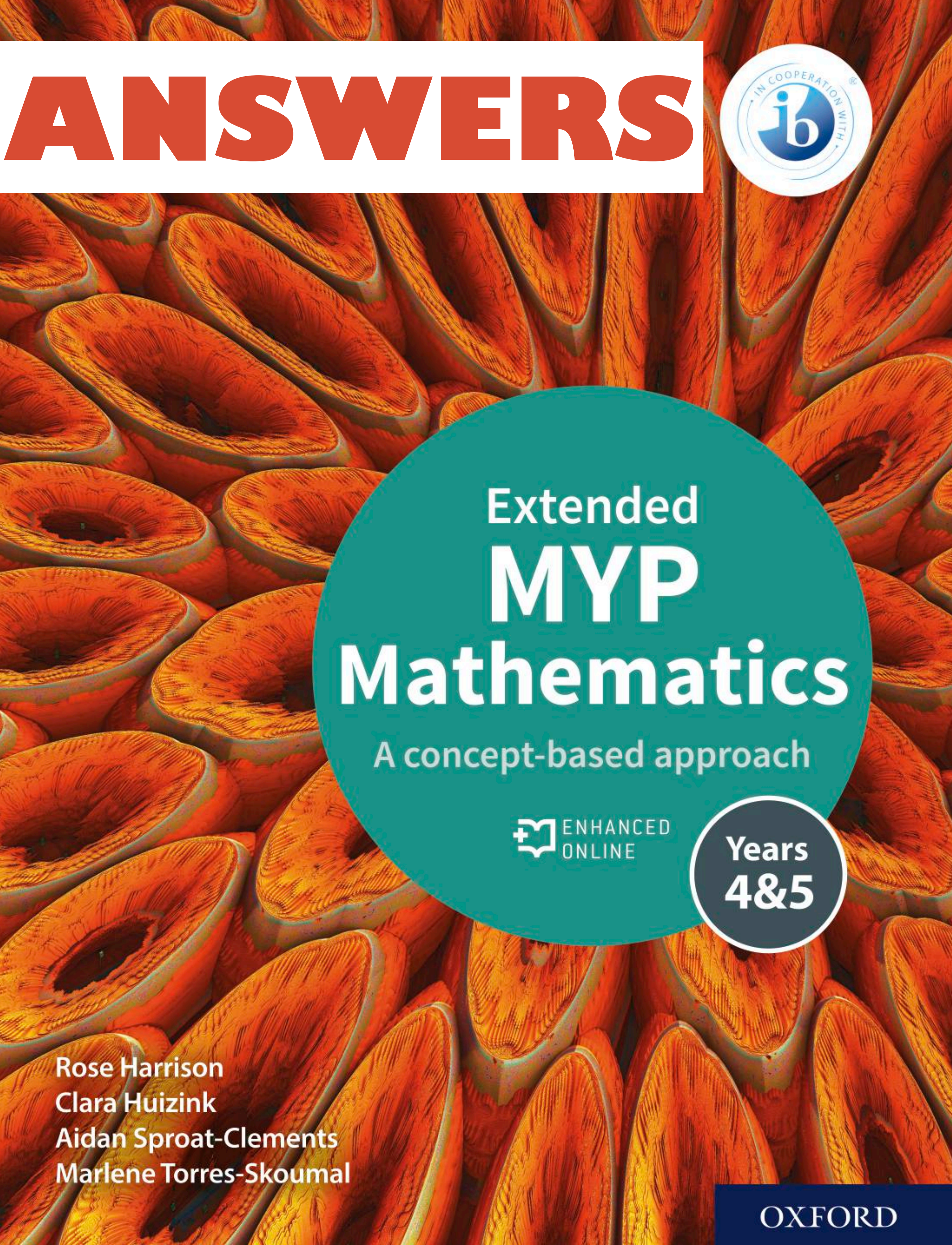

ANSWERS

Extended
MYP Mathematics
A concept-based approach

ENHANCED ONLINE

Years 4&5

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OXFORD

Unit 1 Answers

E1.1

You should already know how to:

1 a x^6 **b** 2^8 **c** $8x^6$

2 a $3\sqrt{5}$ **b** 4 **c** $6\sqrt{5}$

3 a $\frac{1}{4}$ **b** $\frac{1}{36}$ **c** $\frac{1}{8}$ **d** 2

e $\frac{3}{2}$ **f** $\frac{25}{4}$

Practice 1

1 a $\frac{1}{2}$ **b** 2 **c** $\frac{1}{2}$ **d** 5

e $\frac{2}{3}$ **f** $\frac{1}{3}$ **g** $\frac{5}{2}$ **h** $\frac{10}{3}$

2 a $2x^{\frac{1}{2}}$ **b** $3x$ **c** $\frac{1}{2x^2}$ **d** $\frac{10}{x^2}$

e 1 **f** $x^{-\frac{1}{6}}$ **g** $x^{-\frac{1}{4}}$ **h** $3x$

3 a $x^{-\frac{1}{2}}$ **b** $x^{\frac{1}{3}}$ **c** $x^{-\frac{1}{4}}$

4 a $x = 8$ **b** $y = 100$

Practice 2

1 a 4 **b** 81 **c** 125 **d** 16

e 25 **f** 49

2 a $x^{\frac{2}{3}}$ **b** $x^{\frac{3}{4}}$ **c** $x^{\frac{3}{2}}$ **d** $x^{\frac{5}{2}}$

3 a $x^{\frac{4}{3}}$ **b** $x^{\frac{13}{3}}$ **c** $x^{\frac{7}{6}}$ **d** $3^2 = 9$

e $x^{\frac{4}{15}}$ **f** 16

Practice 3

1 a $\frac{1}{125}$ **b** $\frac{1}{8}$ **c** $\frac{64}{27}$ **d** 256

e $\frac{125}{216}$ **f** $\frac{8}{27}$

2 a 5 **b** 2 **c** 2401 **d** 4

e 81 **f** $\sqrt{3}$

3 Student's own answers

4 a $\frac{1}{9}$ **b** 4 **c** 9 **d** 2

Practice 4

1 a $2^{\frac{7}{2}}$ **b** $2^{\frac{7}{3}}$ **c** $2^{\frac{7}{2}}$ **d** $2^{-\frac{5}{3}}$

2 a $7^{\frac{8}{15}}$ **b** $3^{\frac{3}{2}}$ **c** $2^{\frac{1}{4}}$ **d** $2^{\frac{1}{4}}$

3 a $2^{\frac{1}{2}} \times 3^{\frac{1}{2}}$ **b** $2^{\frac{1}{3}} \times 7^{\frac{1}{3}}$ **c** $3 \times 2^{\frac{1}{4}}$ **d** $2^{\frac{1}{2}} \times 5^{\frac{1}{2}}$

e $2^{\frac{1}{4}} \times 5^{\frac{1}{2}}$ **f** $2^{\frac{1}{2}} \times 3^{\frac{1}{3}}$

4 a $2^{\frac{5}{6}}$ **b** $3^{\frac{7}{6}}$ **c** $7^{\frac{3}{10}}$ **d** $2^{\frac{1}{6}}3^{\frac{1}{6}}$

e $2^{\frac{4}{15}}$ **f** $2^{\left(\frac{1}{n}-\frac{1}{m}\right)} \times 5^{\left(\frac{1}{n}-\frac{1}{m}\right)} = 2^{\left(\frac{m-n}{mn}\right)} \times 5^{\left(\frac{m-n}{mn}\right)}$

g $a^{\left(\frac{1}{n}+\frac{1}{m}\right)} = a^{\left(\frac{m+n}{mn}\right)}$ **h** $a^{\left(\frac{1}{n}-\frac{1}{m}\right)} = a^{\left(\frac{m-n}{mn}\right)}$

Practice 5

1 a 2 **b** 3 **c** $\sqrt[6]{5}$ **d** $\frac{1}{\sqrt[6]{11}}$ **e** 7

f $\sqrt[6]{3^5}$ **g** $15\sqrt[6]{2^5}$ **h** $10\sqrt[4]{3^7}$ **i** $2\sqrt[6]{5}$ **j** $\frac{9}{2}$

2 a $\sqrt[3]{2^5}$ **b** $\sqrt[12]{3^7} \times \sqrt{2}$ **c** $\sqrt[3]{2^4} \times \sqrt[6]{3^7}$ **d** $\sqrt[6]{2^5}$ **e** $\frac{1}{\sqrt[6]{7^7}}$

f $\sqrt[12]{5^5}$ **g** $4\sqrt{2}$ **h** $5\sqrt{5}$ **i** $\frac{3}{2}$ **j** 7

3 a $x = 3$ **b** $y = \sqrt[3]{3}$ **c** $z = \sqrt{3}$

Practice 6

1 a $\sqrt[5]{3^2}$ **b** 32 **c** 1000

d $\sqrt[5]{3^3}$ **e** 8 **f** 1000

2 $32^{0.2} = (32)^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = 2$

3 a 2.4 **b** 1 **c** 4.6

4 $x = 1024$

5 a 3 **b** 2

Mixed practice

1 a $\frac{1}{3}$ **b** 3 **c** $\frac{1}{5}$ **d** 3

e $\frac{3}{4}$ **f** 2 **g** $\frac{5}{2}$

2 a $7x^3$ **b** $\frac{1}{4x}$ **c** $\frac{3}{2x}$

3 a 1000 **b** 100 **c** $\frac{1}{9}$ **d** $\frac{1}{27}$

4 a 4 **b** $\frac{27}{8}$ **c** $\frac{25}{16}$ **d** $\frac{25}{27}$

5 a $2^{\frac{5}{2}}$ **b** $3^{\frac{5}{4}}$ **c** 2 **d** $2^{\frac{7}{3}} \times 3^{\frac{7}{3}}$

6 a 25 **b** $\frac{1}{2}$ **c** 7

7 a $\sqrt[3]{2^7}$ **b** $2\sqrt{6}$ **c** 2 **d** $\sqrt[3]{3}$

e 2 **f** 8 **g** $\frac{1}{\sqrt[12]{12}}$ **h** $\frac{1}{\sqrt[12]{3}}$

8 Student's own answers

9 a $\sqrt[3]{2^7}$ **b** 12 **c** $\frac{1}{\sqrt{2}}$ **d** $\frac{\sqrt[12]{5}}{\sqrt[4]{2^5}}$

10 a $x = \frac{1}{3}$ **b** $x = 2$ **c** $x = 27$

11 a $\sqrt[5]{5^9}$ **b** 5 **c** 9 **d** 1

e $2^6 \times 3^3 = 1728$

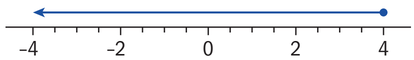
12 Mathematics is as old as man.

E1.2

You should already know how to:

1 a 5200 **b** 23.961 **c** 18.24

2 a



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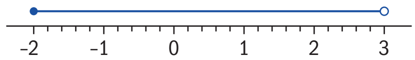

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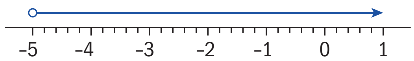


3 a 424.3 **b** 0.001125 **c** 98.231 **d** 5.236

Practice 1

1 1.55 m ≤ length of wood < 1.65 m

2 375 kg ≤ mass of crate < 425 kg

3 59.5 m ≤ length of rope < 60.5 m

4 55 km ≤ length of island of Mauritius < 65 km

5 It is acceptable since 'within 3 mm of 7 cm' means that 67 mm < h < 73 mm.

6 The card may not fit in the envelope. 9.45 cm ≤ length of card < 9.55 cm and 9.5 cm ≤ length of envelope < 10.5 cm. The card will not fit if its length is greater than the length of the envelope, which happens when 9.5 cm ≤ length of envelope < length of card < 9.55 cm

7 21.5 kg ≤ mass < 22.5 kg
12.55 cm ≤ length < 12.65 cm
2.035 cm^3 ≤ volume < 2.045 cm^3
0.65 tonnes ≤ mass < 0.75 tonnes
132.5 cm ≤ height < 133.5 cm
51 500 m^2 ≤ area < 52 500 m^2
58.65° ≤ temperature < 58.75°

Practice 2

1 34 boxes

2 34 cm ≤ perimeter of photoframe < 34.4 cm

3 59.9 m ≤ perimeter of garden < 62.1m, and 193.725 m^2 ≤ area of garden < 205.325 m^2

4 25 minutes

5 She can fill between 10 and 12 full glasses.

Mixed practice

1 a $35 \le x < 45$
b $65 \le x < 75$
c $115 \le x < 125$

2 a $1.5 \le x < 2.5$
b $16.5 \le x < 17.5$
c $-85.5 \le x < -84.5$

3 a 12.45 cm ≤ x < 12.55 cm
b 21.65 cm ≤ x < 21.75 cm
c 34.75 cm ≤ x < 35.85 cm
d 52.05 cm ≤ x < 52.15 cm
e 80.35 cm ≤ x < 80.45 cm

4 a 458 m ≤ perimeter < 462 m
b 12 210.25 m^2 ≤ area < 12 440.25 m^2

5 3.76 cm ≤ side length ≤ 4.26 cm

6 12.16 km/liter

7 0.027

8 a 14.6 **b** 1.8
c 52.4 **d** 1.28
e 14.8 **f** 2
g 53.9 **h** 1.31

Review in context

1 a 6850 km ≤ radius of satellite's orbit < 6900 km
b 43 332 km
c 314 km

2 22 fuel tanks are needed, since it needs a minimum of 21.66 fuel tanks (21 is therefore not enough).

3 a 229 days (the actual time is 228.4 days, but it would not have reached Mars after only 228 days).
b 252.9 days rounded to 253 days.

Unit 2 Answers

E2.1

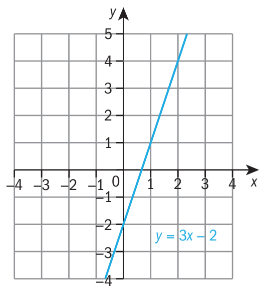
You should already know how to:

1 **a** $6 < 8$, valid **b** $2 < 4$, valid
  c $6 < 12$, valid **d** $-3 < -6$, not valid

2 **a** $x = 2$, addition and multiplication principles.
  b $x = 12$, addition and multiplication principles.

3 **a** $x > 3$ **b** $x < 2$
  c $x \geq 3$ **d** $-2 < x < 2$

4 **a**



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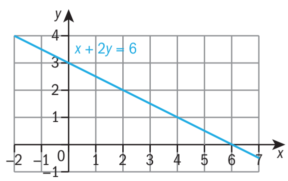


Practice 1

1 $x < 4$ **2** $x > 54$
3 $m < 1$ **4** $x \geq 8$
5 $x \geq 2$ **6** $x < -\frac{7}{17}$
7 $a > 4$ **8** $b \leq 1.25$
9 $k \leq 6.4$

Practice 2

1 Solutions the same as in **Practice 1**

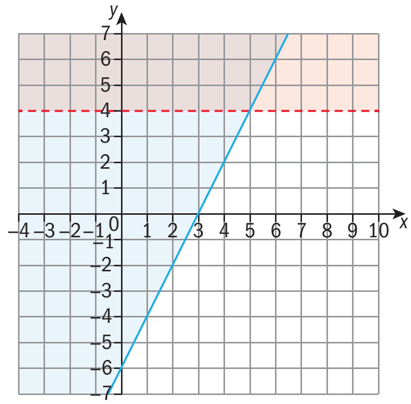
2 $y \geq 6 - x$ and $y > 4$

3 $3x + 2 < 9x + 6$
$-6x < 4$
$x > \frac{-2}{3}$
which is an infinite set of solutions.

Practice 3

1 The unshaded area is the solution set.

a



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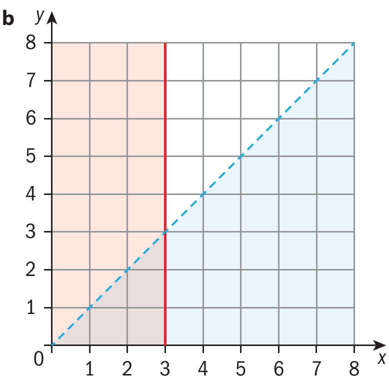
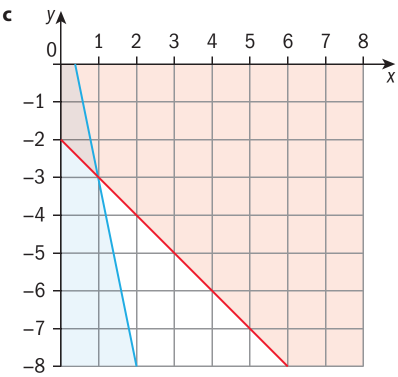
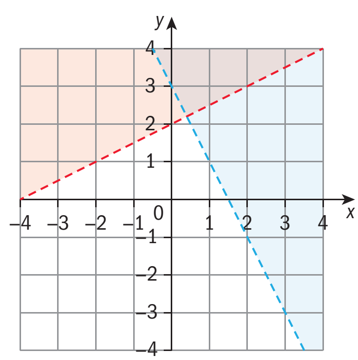


c



d



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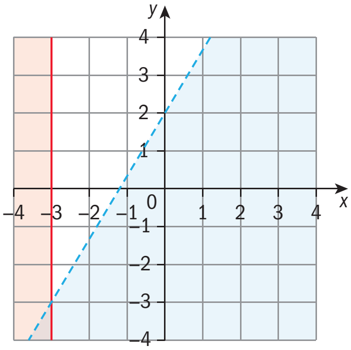

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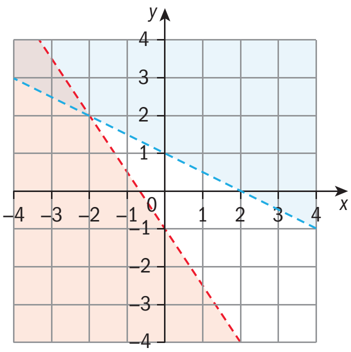

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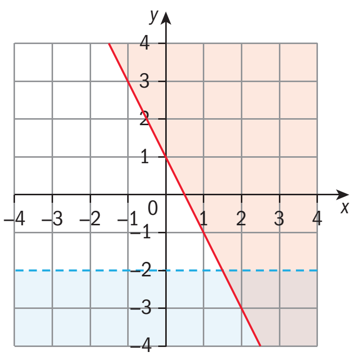

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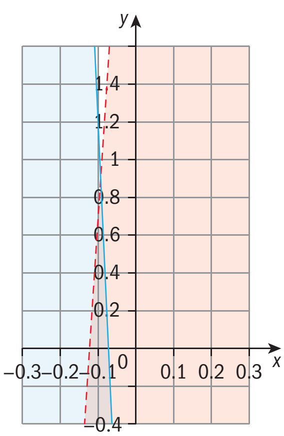
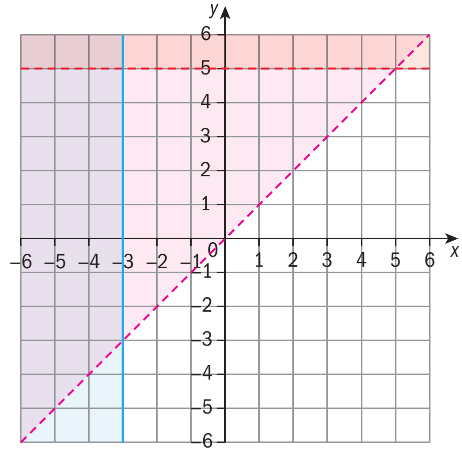


2 $x < 6$; $y \leq x$; $y \geq 6 - 2x$

3 The unshaded area is the solution set.

a



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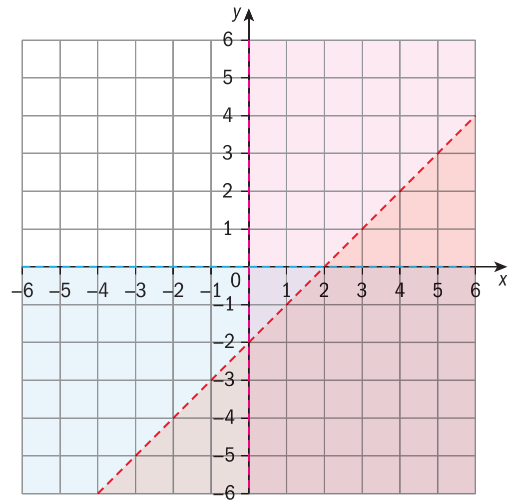
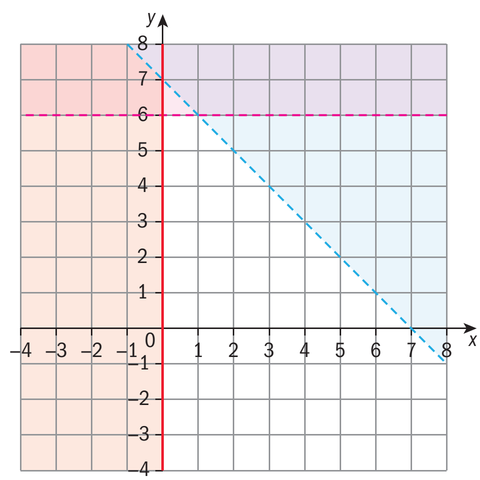


4 The unshaded area is the solution set.

a



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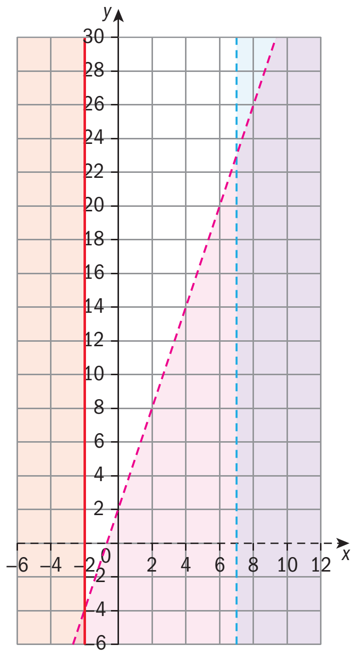

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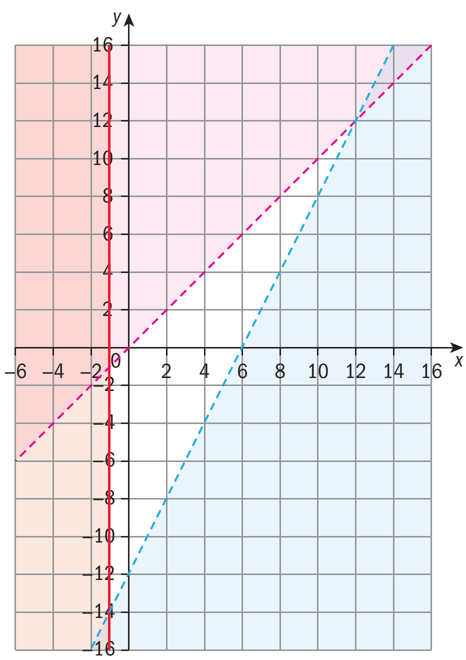

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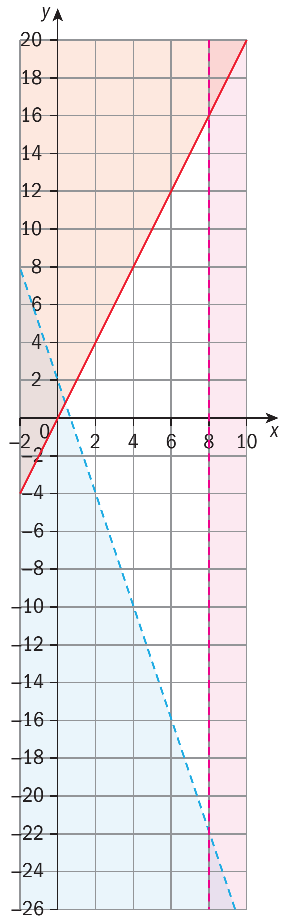


Practice 4

1. Maximum = 780 at (15, 1.5)
2. Maximum = 19.6 at (1, 6)
3. Maximum = 707 at (99, 181)
4. The manufacturer should make 105 mid-top shoes and 45 high-top shoes, which would make a profit of $2085 per day.
5. 6 small and 6 large minibuses
6. 100 algebraic solvers and 170 graphing programs
7. $640 (from 40 downhill skis and 30 cross-country skis)
8. $75 000 in municipal bonds and $25 000 in bank mutual fund

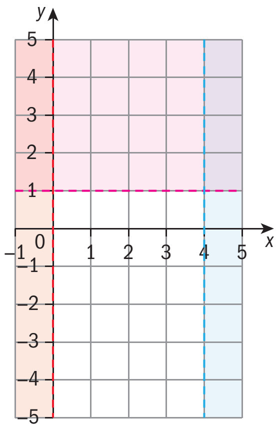
Mixed practice

1 **a** $x \geq 1$
b $x < 18$
c $x > -9$
d $x < -\frac{1}{5}$
e $x \geq 4$
f $5 \leq x \leq 9$
g $\frac{1}{2} < x < 8$

2 Solutions same as **1 a–d**

3 The unshaded area is the solution set.

a



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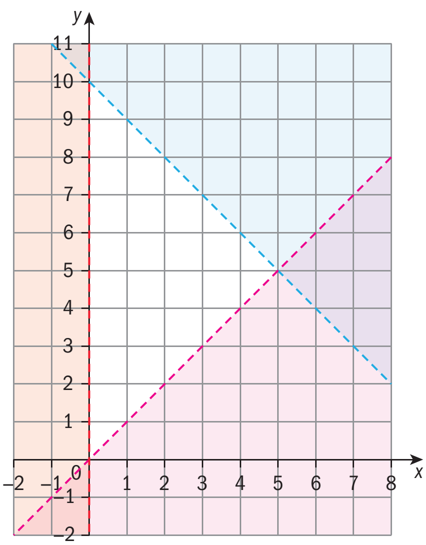

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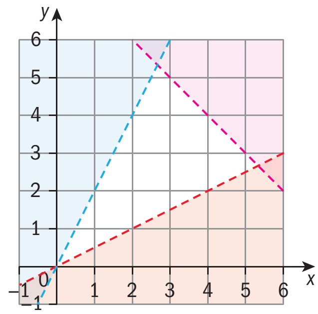

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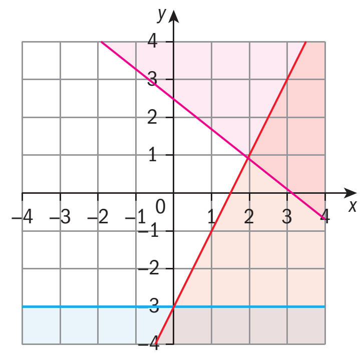


4 Three 40-seater and no 24-seater buses

5 The factory can meet its target.
It can buy 8 Machine A and 0 Machine B, or 6 Machine A and 3 Machine B.

6 12 weekday and 8 weekend advertisements

Review in context

1 6 large trees and 20 small trees

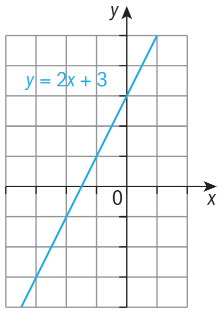
2 21 type A and 8 type B costing \$83

Unit 3 Answers

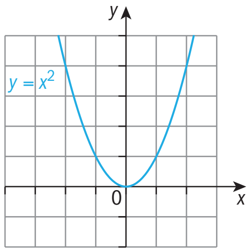
E3.1

You should already know how to:

1 a



b



2 a $(x+3)^2-1$ **b** $(-3, -1)$

Practice 1

1 a horizontal translation of 4 units
b horizontal dilation of scale factor $\frac{1}{3}$
c reflection in the x-axis
d reflection in the y-axis
e vertical dilation of scale factor 4
f horizontal translation of 3 units and vertical translation of 4 units

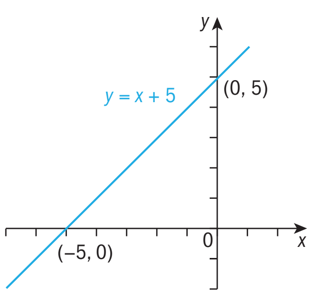
2 a vertical dilation of scale factor 2, vertical translation of −3 units
b horizontal dilation of scale factor $\frac{1}{2}$, reflection in the x-axis
c horizontal translation of 4 units, vertical dilation of scale factor $\frac{1}{2}$
d reflection in the x-axis and vertical translation of 4 units

3 Both are correct. The transformation from $f(x)=(x-2)^2$ to $g(x)=(x+2)^2$ can be seen as a horizontal translation of −4 units ($g(x)=f(x+4)$) or a reflection in the y-axis ($g(x)=f(-x)$).

Practice 2

1 a A: $y=-3x-3$ B: $y=-3x-1$ C: $y=-3x+4$
b D: $y=0.5x$ E: $y=0.5x-2$ F: $y=0.5x-3$
c G: $y=x^2-3$ H: $y=(x-4)^2$ J: $y=(x-7)^2-4$
d K: $y=-0.5x^2-4$ L: $y=-0.5(x+7)^2$ M: $y=-0.5(x+2)^2-3$

2 a



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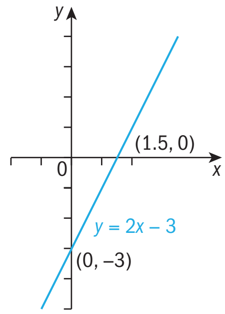

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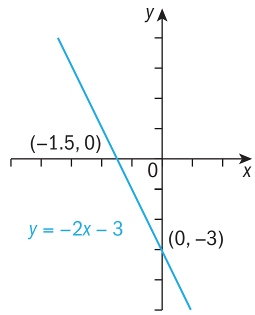

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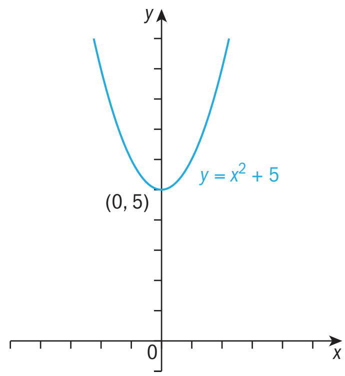

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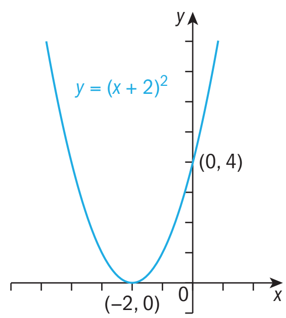

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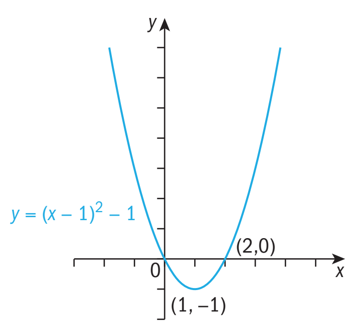
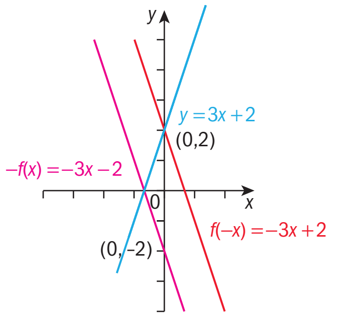


Practice 3

1



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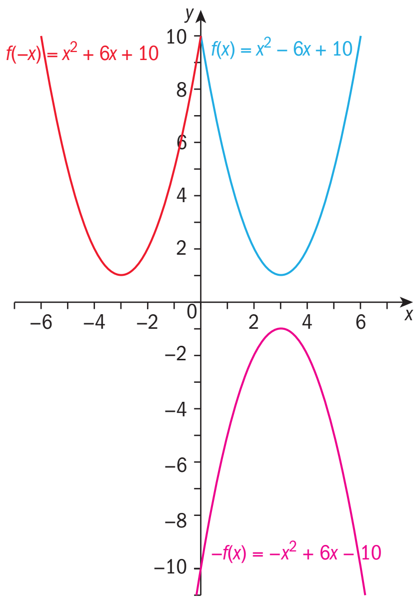


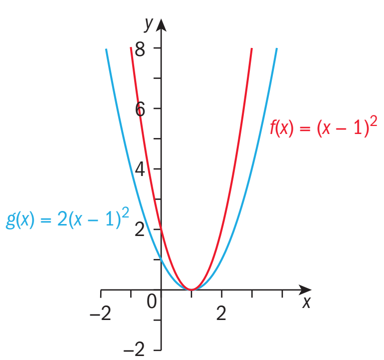
3 **a** L_1: $y = x + 1$ L_2: $y = x - 1$
b L_3: $y = x^2 + 4x + 4$ L_4: $y = -x^2 + 4x - 4$
c L_5: $y = (x + 3)^2 + 1$ L_6: $y = -(x - 3)^2 - 1$
d L_7: $y = (x - 3)^2 - 2$ L_8: $y = -(x + 3)^2 + 2$

4 $y = -x + 4$

5 Curve C: $y = -(x - 2)^2$

Practice 4

1



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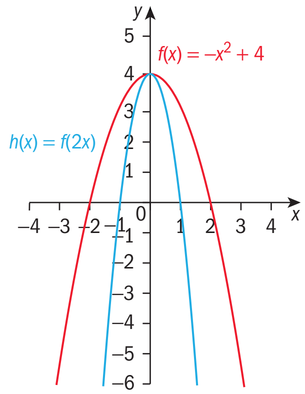

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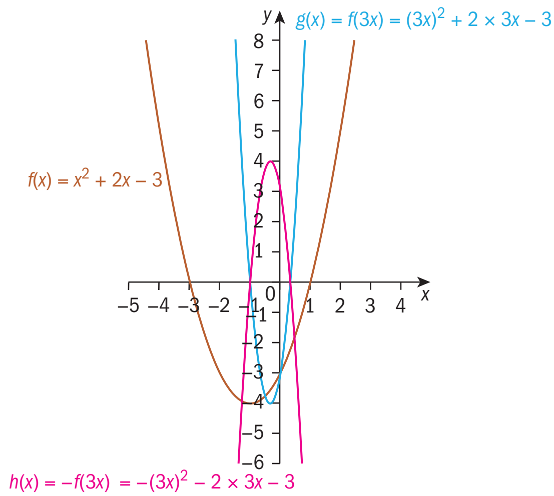

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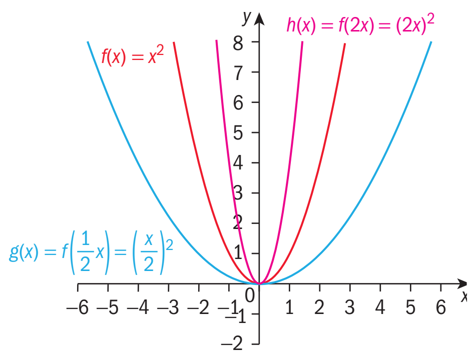


5 a Dilation scale factor 3 parallel to the y-axis
b Dilation scale factor $\frac{1}{2}$ parallel to the y-axis
c Dilation scale factor $\frac{1}{4}$ parallel to the x-axis
d Dilation scale factor 3 parallel to the x-axis

6 a $y = 5x^2$ **b** $y = \left(\frac{x}{4}\right)^2$ **c** $y = \frac{x^2}{3}$

7 a Dilation scale factor 3 parallel to the y-axis
b Dilation scale factor 2 parallel to the x-axis

Practice 5

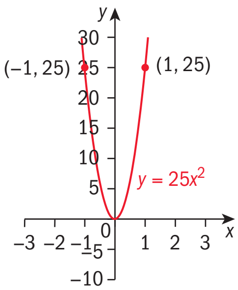
1 a Dilation scale factor 4 parallel to the *y*-axis and reflection in *x*-axis

b Dilation scale factor 2 parallel to the *y*-axis and vertical translation of –9 units

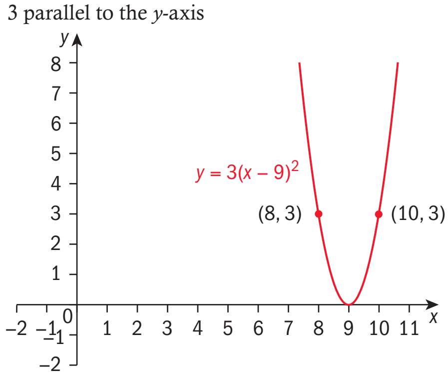
c Dilation scale factor 0.5 parallel to the *y*-axis and vertical translation of 4 units

d Dilation scale factor 1.2 parallel to the *y*-axis, reflection in *x*-axis and vertical translation of –2.5 units

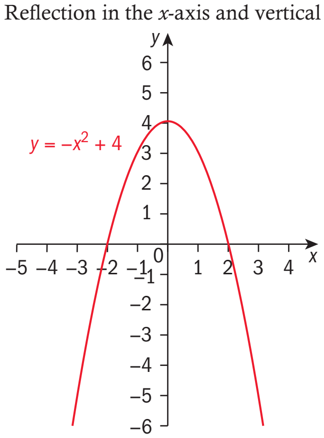
2 a Dilation scale factor 25 parallel to the *y*-axis



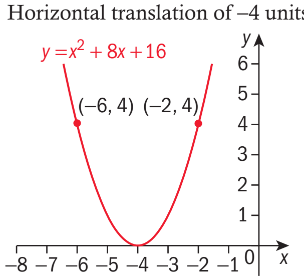
b Horizontal translation of 9 units then dilation scale factor 3 parallel to the *y*-axis



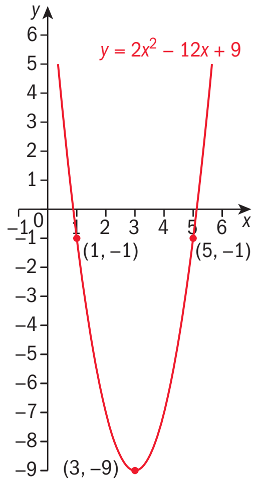
c Reflection in the *x*-axis and vertical translation of 4 units



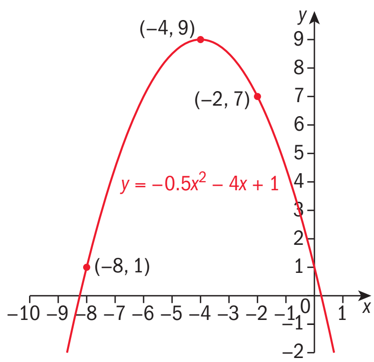
d Horizontal translation of –4 units



e Dilation scale factor 2 parallel to the *y*-axis and horizontal translation of 3 units, vertical translation of –9 units



f Dilation scale factor $\frac{1}{2}$ parallel to the *y*-axis, reflection in the *x*-axis, horizontal translation of –4 units and vertical translation of 9 units.



3 a $g(x) = 4f(x)$ vertical dilation scale factor 4

b $g(x) = f(3x)$ horizontal dilation scale factor $\frac{1}{3}$

c $g(x) = -f(x)$ reflection in the *x*-axis

d $g(x) = f(-x)$ reflection in the *y*-axis

e $g(x) = f(x) - 12$ vertical translation of –12 units

f $g(x) = f(-x)$ reflection in the *y*-axis

g $g(x) = f(x - 2)$ horizontal translation of 2 units

h $g(x) = \frac{1}{4}f(x)$ vertical dilation of scale factor $\frac{1}{4}$

4 a $g(x) = -f(x) - 5$ or $g(x) = f(-x) + 11$: reflection in the *x*-axis and vertical translation of –5 units

b $g(x) = \left(\frac{1}{2}\right)f(x) - 6$ or $g(x) = f\left(\left(\frac{1}{2}\right)x\right) - 9$: vertical dilation of scale factor $\frac{1}{2}$ and vertical translation of –6 units or horizontal dilation of scale factor 2 and vertical translation of –9 units.

c $g(x) = f(-x) + 4$: reflection in the *y*-axis and vertical translation of 4 units

d $g(x) = -\left(\frac{1}{2}\right)f(x)$: reflection in the *x*-axis and vertical dilation of scale factor $\frac{1}{2}$

e $g(x) = -f(x) + 9$: reflection in the *x*-axis and vertical translation of 9 units

f $g(x) = f(x - 5)$: horizontal translation of 5 units (only 1 transformation)

g $g(x) = f(x + 4) - 1$: horizontal translation of −4 units and vertical translation of −1 unit

h $g(x) = f(-3x)$: reflection in the y-axis and horizontal dilation of scale factor $\frac{1}{3}$

5 **a** $f_2(x) = -f_1(x)$: Reflection in the x-axis

b $f_2(x) = f_1(2x)$: Dilation scale factor $\frac{1}{2}$ parallel to the x-axis

c $f_2(x) = 2f_1(x)$: Dilation scale factor 2 parallel to the y-axis

d $f_2(x) = -f_1(x)$: Reflection in the x-axis

e $f_2(x) = f_1(x + 4)$: Horizontal translation of −4 units

f $f_2(x) = \frac{1}{2}f_1(x)$: Dilation scale factor $\frac{1}{2}$ parallel to the y-axis

6 **a** $f_2(x) = -f_1(x) + 3$. Reflect $f_1(x)$ in the x-axis, and vertical translation of 3 units OR $f_2(x) = f_1(-x) - 3$: Reflect $f_1(x)$ in the y-axis and vertical translation of −3 units.

b $f_2(x) = 2f_1(x - 1)$. Horizontal translation of $f_1(x)$ of 1 unit then a vertical dilation of scale factor 2.

c $f_2(x) = -f_1(x + 2)$. Reflect $f_1(x)$ in the x-axis then horizontal translation of −2 units.

Practice 6

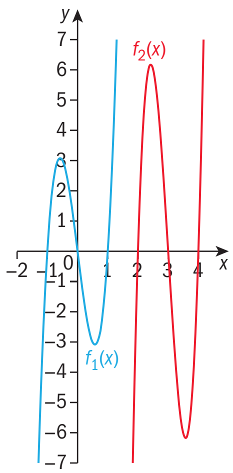
1 **a** $f_2(x) = 2f_1(x + 3)$

b $f_2(x) = \frac{1}{2}f_1(2x)$

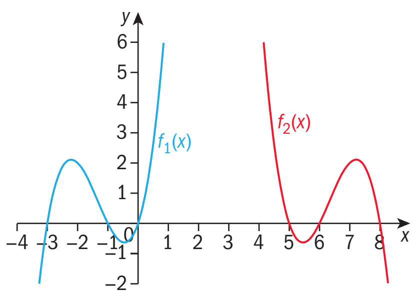
c $f_2(x) = -f_1(x + 4)$

d $f_2(x) = 3f_2\left(\frac{x}{2}\right)$

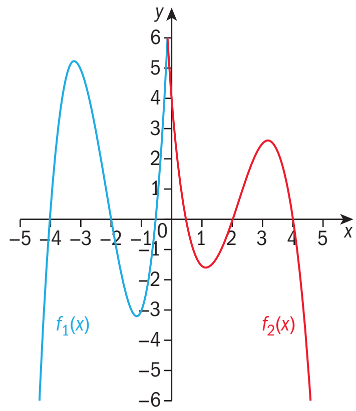
2 **a**



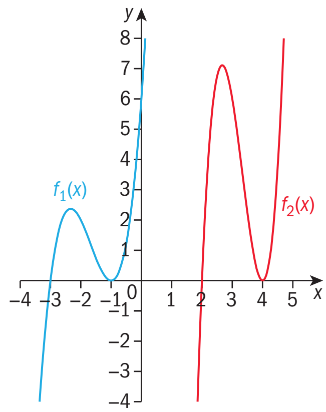
b



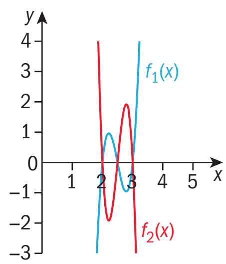
c



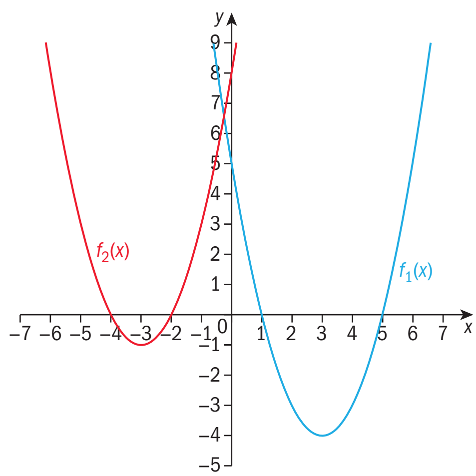
d



e

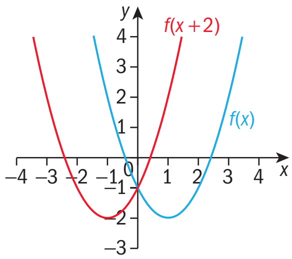


f

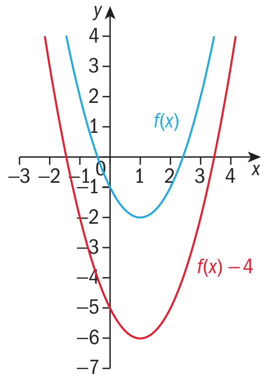


Mixed practice

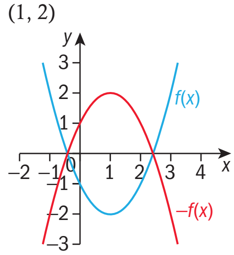
1 a (–1, –2)



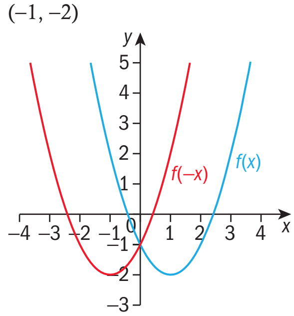
b (1, –6)



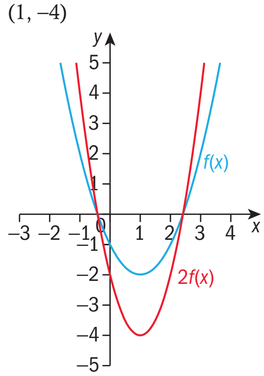
c (1, 2)



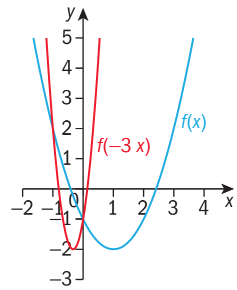
d (–1, –2)



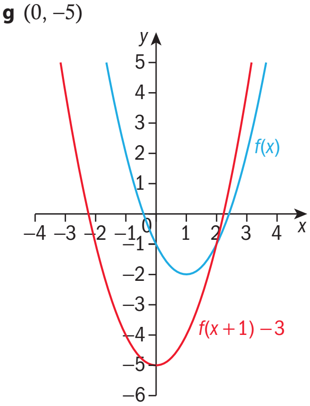
e (1, –4)



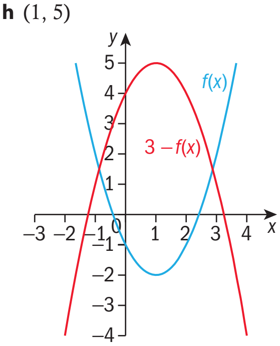
f $\left(-\frac{1}{3}, -2\right)$



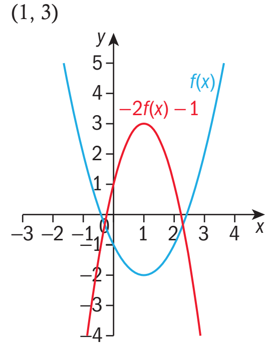
g (0, –5)



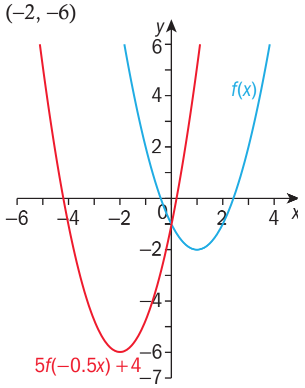
h (1, 5)



i (1, 3)



j (−2, −6)



2 a Dilation scale factor $\frac{1}{2}$ parallel to the x-axis and vertical translation of −8 units OR dilation scale factor 2 parallel to the y-axis and vertical translation of −8 units

b Dilation scale factor 3 parallel to the x-axis, reflection in the y-axis and vertical translation of 8 units OR dilation scale factor $\frac{1}{3}$ parallel to the y-axis and reflection in the x-axis

c Dilation scale factor $\frac{1}{2}$ parallel to the y-axis and vertical translation of 3.5 units OR Dilation scale factor 2 parallel to the x-axis and vertical translation of 1 unit

d Horizontal translation of 2 units and vertical translation of −5 units

e Horizontal translation of −3 units and vertical translation of −3 units

f Dilation scale factor $\frac{1}{2}$ parallel to the y-axis and vertical translation of 2 units

3 a Reflection in the y-axis: $f_2(x) = f_1(-x)$

b Horizontal dilation scale factor 2: $f_2(x) = f_1\left(\frac{x}{2}\right)$

c Horizontal translation −3 units: $f_2(x) = f_1(x + 3)$

d Reflection in the x-axis: $f_2(x) = -f_1(x)$

e Vertical dilation scale factor $\frac{1}{2}$: $f_2(x) = 0.5f_1(x)$

f Horizontal dilation scale factor $\frac{1}{2}$: $f_2(x) = f_1(2x)$

4 a Reflection in the x-axis and vertical translation of 3 units $f_2(x) = -f_1(x) + 3$ or reflection in the y-axis and vertical translation of 5 units: $f_2(x) = f_1(-x) + 5$

b Reflection in the x-axis and vertical translation of −1 unit $f_2(x) = -f_1(x) - 1$

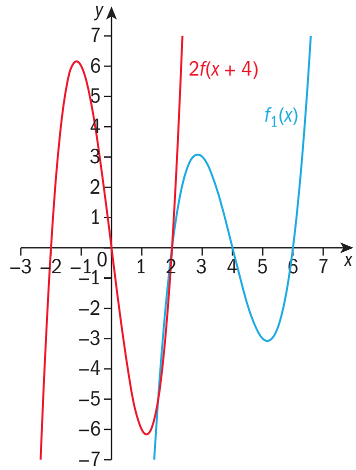
c Reflection in the x-axis and horizontal translation of −6 units $f_2(x) = -f_1(x + 6)$

d Vertical translation of 4 units and horizontal translation of 4 units $f_2(x) = f_1(x - 4) + 4$

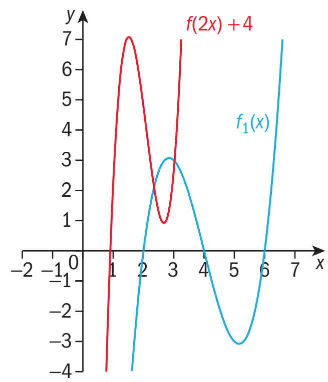
e Vertical dilation of scale factor 2 and horizontal translation of 3 units $f_2(x) = 2f_1(x - 3)$

f Reflection in the x-axis and vertical translation of −2 units $f_2(x) = -f_1(x) - 2$

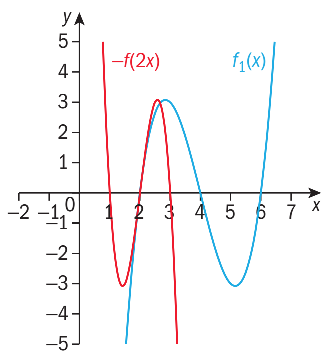
5 a



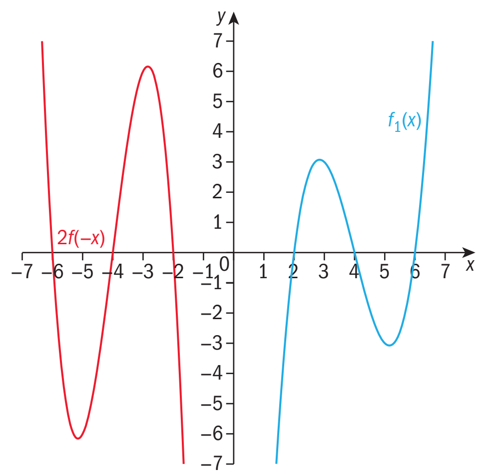
b



c

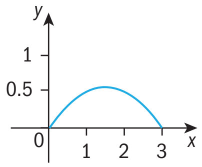


d



Review in context

1 a



b 3 m

c i It would reach the same height but it would only travel half as far horizontally.

ii It would reach a greater maximum height, 0.675 m, and a greater overall distance, 4.29 m.

iii The frog has jumped to the left, not to the right.

d Because this would suggest that the frog has jumped downwards, then returned back upwards to its initial starting height.

2 a It translates the graph upwards – the ball would have been thrown from a greater height.

b It stretches the graph horizontally – the ball would have been thrown at a greater speed.

3 a i Horizontal translation of 10 units, reflect in the x-axis, dilate by a scale factor of $\frac{5}{9}$ in the y direction and translate by 5 units in the y direction.

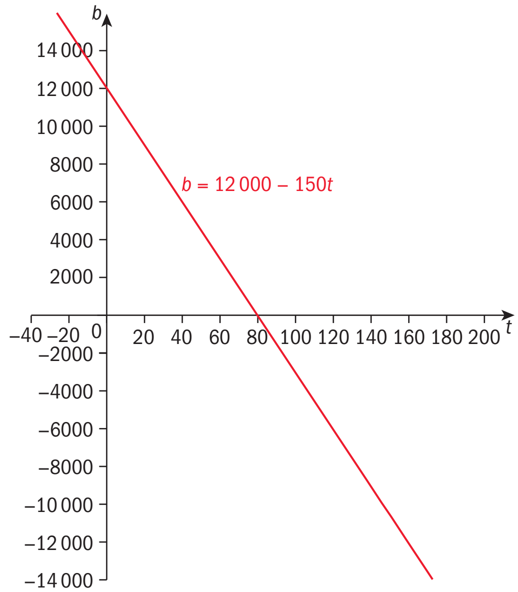
ii $-\frac{1}{20}(0-10)^2+5=-\frac{1}{20}\times 100+5=-5+5=0.$

iii Minimum point of x^2 is at (0,0) and is mapped to a maximum point at (10,5).
It would hit the ceiling.

b The coordinates of the maximum point are (10, 2.5). With a maximum height of 2.5 m, the ball would not hit the ceiling.

4 a $b(t) = 12\,000 - 150t$

b



c 4500 – the loan balance after 50 months.

d 80 months

e A larger loan

f A smaller loan and smaller monthly repayments

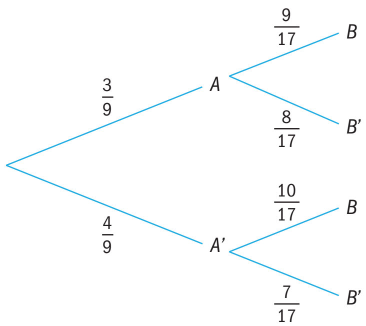
g It will be steeper with the same y-intercept. $b(t) = 12000 - 250t$

Unit 4 Answers

E4.1

You should already know how to:

1 a A = Event that first song is from the 1980s
B = Event that second song is from the 1980s



b $\frac{5}{9} \times \frac{9}{17} = \frac{5}{17}$

2 a $\frac{8}{13}$

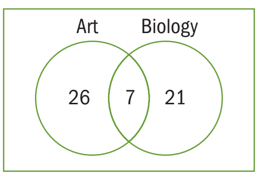
b $\frac{4}{52} \times \frac{12}{52} = \frac{3}{169}$

c $\frac{4}{52} \times \frac{12}{51} = \frac{4}{221}$

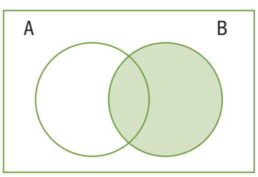
3 a $\frac{5}{9}$

b 1

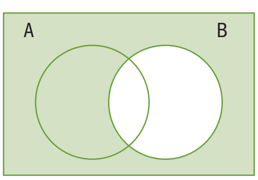
4



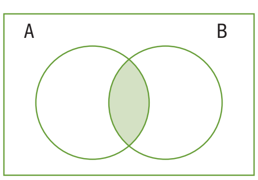
5 a



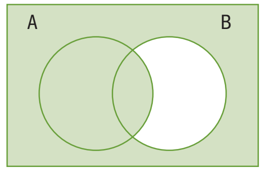
b



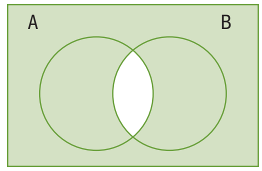
c



d



e



Practice 1

1 a $\frac{1}{24}$ **b** Yes since $P(A) \times P(B) = P(A \cap B)$

c $\frac{1}{4} \times \frac{1}{6} = \frac{1}{24}$

2 a $\frac{1}{18}$ **b** Yes since $P(A) \times P(B) = P(A \cap B)$

c $\frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$

3 a 28

b Yes since the first domino chosen is replaced

c $\frac{7}{28} \times \frac{7}{28} = \frac{1}{16}$

4 $\frac{9}{20} \times \frac{10}{20} \neq \frac{6}{20}$ so no, not independent

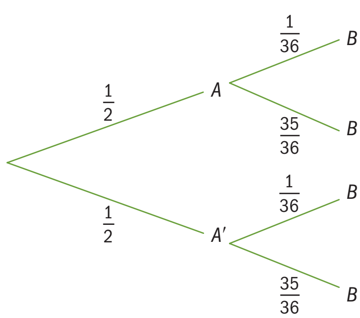
5 $\frac{6}{24} \times \frac{8}{24} = \frac{2}{24}$ so yes, independent

6 a $\frac{39}{90} = \frac{13}{30}$ **b** $\frac{60}{90} = \frac{2}{3}$ **c** $\frac{35}{90} = \frac{7}{18}$

d Not independent since P(trampolining) × P(table tennis) ≠ 0

Practice 2

1



$P(B \mid A) = \frac{1}{36}$

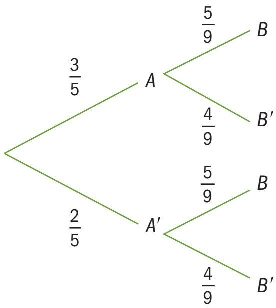
$P(B \mid A') = \frac{1}{36}$

$P(B) = \frac{1}{72} + \frac{1}{72} = \frac{1}{36}$

$P(B \mid A) = P(B \mid A') = P(B)$

∴ independent events

2



$P(B \mid A) = \frac{5}{9}$

$P(B \mid A') = \frac{5}{9}$

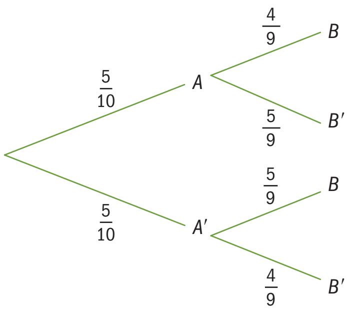
$P(B) = P(A \cap B) + P(A' \cap B)$

$= \frac{3}{9} + \frac{2}{9} = \frac{5}{9}$

$P(B \mid A) = P(B \mid A') = P(B)$

$\therefore$ independent events

3



$P(B \mid A) = \frac{4}{9}$

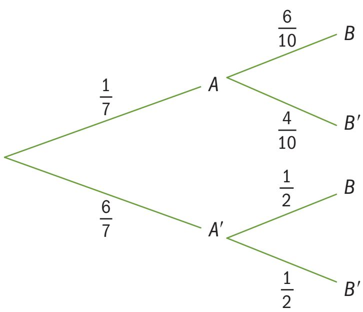
$P(B \mid A') = \frac{5}{9}$

$P(B) = \frac{2}{9} + \frac{5}{18} = \frac{1}{2}$

$P(B \mid A) \neq P(B \mid A') \neq P(B)$

The events are not independent.

4



$P(B \mid A) = \frac{6}{10}$

$P(B \mid A') = \frac{1}{2}$

The events are not independent.

5 $P(B \mid A) = \frac{5}{9}$

$P(B \mid A') = \frac{6}{9}$, therefore not independent events.

6 $P(B \mid A) = \frac{5}{11}$

$P(B \mid A') = \frac{6}{11}$, therefore not independent events.

7 $P(B \mid A) = 90\%$

$P(B \mid A') = 65\%$, therefore not independent events.

8 $P(Y \mid X) = 0.8$

$P(Y \mid X') = 0.8$

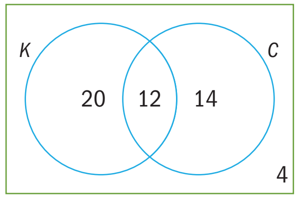
$\therefore P(Y \mid X') = P(Y \mid X) = P(Y)$

$\therefore$ independent events

Practice 3

1 a $\frac{11}{26}$ **b** $\frac{11}{32}$

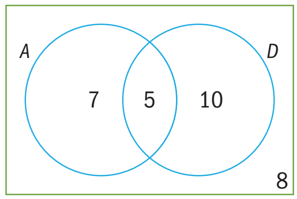
2 a



b i $P(K) = \frac{32}{50} = \frac{16}{25}$ **ii** $P(C \mid K) = \frac{12}{32} = \frac{3}{8}$

iii $P(C) = \frac{26}{50} = \frac{13}{25}$ **iv** $P(K \mid C) = \frac{12}{26} = \frac{6}{13}$

3 a



b i $P(A) = \frac{12}{30} = \frac{2}{5}$ **ii** $P(D \mid A) = \frac{5}{12}$

iii $P(D \cup A) = \frac{22}{30} = \frac{11}{15}$ **iv** $P(D \cap A \mid D \cup A) = \frac{5}{22}$

4 a $\frac{3}{5}$ **b** $\frac{9}{25}$ **c** $\frac{4}{9}$

5 a i $\frac{17}{35}$ **ii** $\frac{19}{35}$ **iii** $\frac{17}{35}$

b i $\frac{9}{19}$ **ii** $\frac{13}{17}$ **iii** $\frac{7}{18}$

c $\frac{1}{4}$

Practice 4

1 a

	Male (*M*)	Female (*F*)	Total
Professional (*P*)	5	12	17
Amateur(*A*)	5	6	11
Total	10	18	28

b i $P(M \mid P) = \frac{5}{17}$ **ii** $P(M \mid A) = \frac{5}{11}$ **iii** $P(M) = \frac{10}{28} = \frac{5}{14}$

c No they are not independent events

2

	Soccer	No soccer	Total
Left	10	30	40
Right	15	45	60
Total	25	75	100

$P(L \mid S) = \frac{10}{25} = \frac{2}{5}$

$P(L \mid NS) = \frac{30}{75} = \frac{2}{5}$

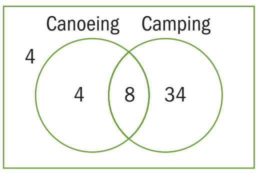
$P(L) = \frac{2}{5}$

They are independent events.

3 Yes, because $P(F \text{ and } S) = P(F) \times P(S)$

Practice 5

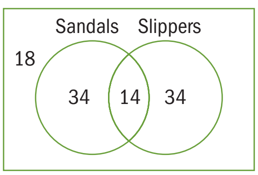
1 a



b 4 **c** $\frac{12}{50} = \frac{6}{25}$ **d** $\frac{12}{50} \times \frac{42}{50} \neq \frac{8}{50}$ so no

e Axiom 2, Axiom 3, Theorem 1, Theorem 2

2 a



b 48 **c** $\frac{14}{48} = \frac{7}{24}$

d i No **ii** No

e Axiom 2, Axiom 3, Proposition 1, Theorem 1, Theorem 2

3

	Right	Left	Total
Male	58	12	70
Female	27	3	30
Total	85	15	100

a $\frac{3}{30} = \frac{1}{10}$

b Yes – a person is left or right handed, not both.

c No since $P(L) \times P(R) \neq 0$

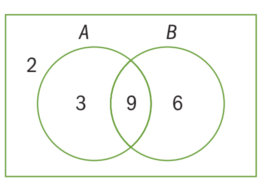
d Axiom 2, Axiom 3, Proposition 1, Theorem1, Theorem 2

Practice 6

1 a i 20 **ii** 12 **iii** 15

iv 18 **v** $\frac{12}{20} = \frac{3}{5}$ **vi** $\frac{15}{20} = \frac{3}{4}$ **vii** $\frac{18}{20} = \frac{9}{10}$

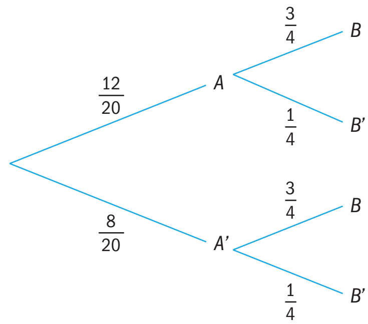
b i $\frac{9}{20}$ **ii**



c i $\frac{9}{12} = \frac{3}{4}$ **ii** $\frac{3}{12} = \frac{1}{4}$

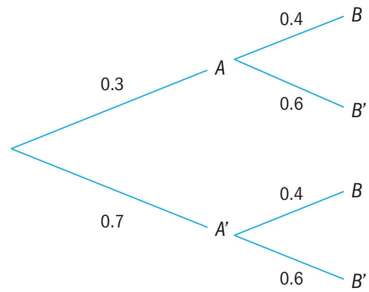
d i $\frac{6}{8} = \frac{3}{4}$ **ii** $\frac{2}{8} = \frac{1}{4}$

e



2 a i 0.7 **ii** 0.12

b

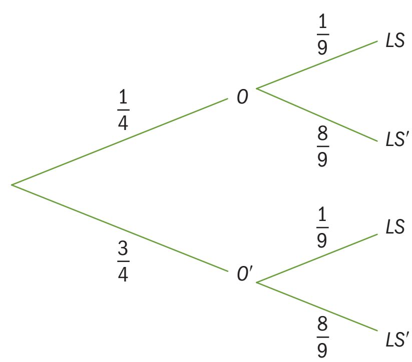


Practice 7

1 a $P(LS \mid \text{operation}) = P(LS \mid \text{No operation}) = P(LS) = \frac{1}{10}$

The results are independent events so the claim is not valid.

b



It is evident from the tree diagram $(LS \mid \text{operation}) = P(LS \mid \text{No operation})$

c Student's own answer with justification.

2 a

	T	T'	Total
G	10	15	25
G'	30	10	40
Total	40	25	65

b The events are not independent.

c Student's own answer

3 a i $P(E) = 0.4$ **ii** $P(L \mid E) = 0.8$

iii $P(L \mid E') = 0.4$ **iv** $P(E \cap L) = 0.32$

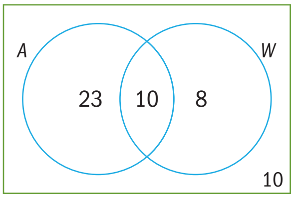
v $P(E \cap L') = 0.08$ **vi** $P(E' \cap L) = 0.24$

vii $P(E' \cap L') = 0.36$ **viii** $P(L) = 0.56$

b No, because the conditional probabilities are different.

Practice 8

1 a



b i $P(A) = \frac{33}{51} = \frac{11}{17}$ **ii** $P(W) = \frac{18}{51} = \frac{6}{17}$

iii $P(A \mid W) = \frac{10}{18} = \frac{5}{9}$ **iv** $P(W \mid A) = \frac{10}{33}$

v $P(W \mid A') = \frac{8}{18} = \frac{4}{9}$

c

	A	A′	
W	10	8	18
W′	23	10	33
	33	18	51

d Student's own verifications

e If A and W are independent events then by Theorem 1 $P(A \cap W) = P(A) \times P(W) = \frac{33}{51} \times \frac{18}{51} = \frac{66}{289}$. Since $P(A \cap W) = \frac{10}{51}$, and $\frac{10}{51} \neq \frac{66}{289}$, A and W are not independent events. If A and W are independent events then by Theorem 2, $P(W \mid A) = P(W \mid A') = P(W)$. Since $P(W \mid A) = \frac{10}{23}$; $P(W \mid A') = \frac{8}{18}$; $P(W) = \frac{18}{51}$, events A and W are not independent.

2 a $\frac{12}{30} = \frac{2}{5}$ **b** $\frac{12}{70} = \frac{6}{35}$

c $P(M)$ does not equal $P(M \mid FT)$ does not equal $P(M \mid FT')$, therefore not independent by Theorem 2.

3 a 88% **b** $\frac{1}{3}$

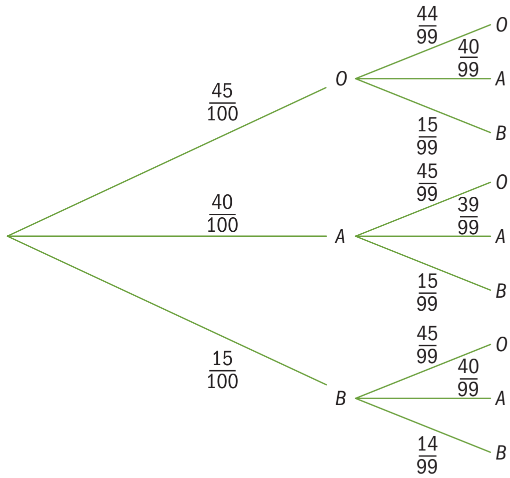
c No, from the tree diagram the second branches are not the same.

4 a $\frac{14}{30} = \frac{7}{15}$ **b** $\frac{14}{70} = \frac{1}{5}$

c Yes they are independent as $P(M \mid L) = \frac{16}{30} = \frac{8}{15} = P(M \mid L') = \frac{64}{120} = \frac{8}{15} = P(M) = \frac{80}{150} = \frac{8}{15}$

Mixed practice

1 a

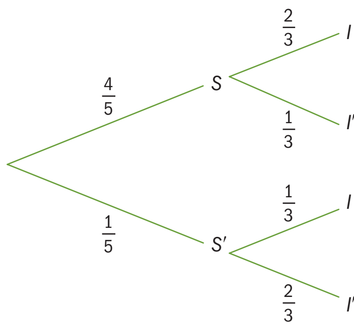


b $\frac{3750}{9900} = \frac{25}{66}$ **c** $\frac{40}{99}$ **d** $\frac{55}{99}$

2 a $\frac{6}{13}$ **b** $\frac{2}{13}$ **c** $\frac{2}{5}$ **d** $\frac{1}{3}$

e $\frac{5}{13}$ **f** $\frac{1}{2}$ **g** $\frac{3}{7}$

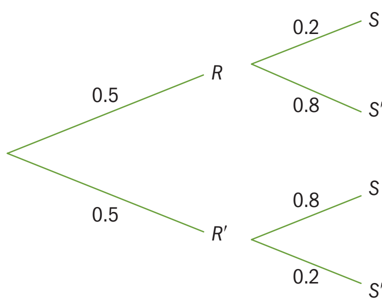
3 a



b $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$

c $P(I \mid S) \neq P(I \mid S')$
$\therefore$ not independent

4 a



b Th 2: $P(R \cap S) = 0.1$

$P(R) = 0.5$ $\quad P(R \cap S) \neq P(R) \times P(S)$

$P(S) = 0.1 + 0.4$

$= 0.5$ $\quad \therefore$ Not independent

Th 3: $P(S \mid R) = 0.2$

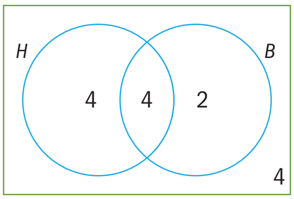
$P(S \mid R') = 0.8$ $\quad \therefore$ Not independent

5 a

	T	*T'*	
B	19	29	48
B'	11	11	22
	30	40	70

b $\frac{3}{7}$ **c** $\frac{29}{48}$ **d** Not independent, by Th 3

6 a



b **i** $\frac{8}{14} = \frac{4}{7}$ **ii** $\frac{6}{14} = \frac{3}{7}$ **iii** $\frac{10}{14} = \frac{5}{7}$

iv $\frac{4}{14} = \frac{2}{7}$ **v** $\frac{6}{14} = \frac{3}{7}$ **vi** $\frac{8}{14} = \frac{4}{7}$

vii $\frac{12}{14} = \frac{6}{7}$ **viii** $\frac{4}{14} = \frac{2}{7}$ **ix** $\frac{4}{14} = \frac{2}{7}$

c **i** $P(H \mid B) = \frac{4}{6} = \frac{2}{3}$ **ii** $P(H \mid B') = \frac{4}{8} = \frac{1}{2}$

iii $P(B \mid H) = \frac{4}{8} = \frac{1}{2}$ **iv** $P(B \mid H') = \frac{2}{6} = \frac{1}{3}$

7 a

	Should have dress code	Should not have dress code	Total
Middle school	15	30	45
High school	40	80	120
Total	55	110	165

b Theorem 2

$P(MS \text{ and dress code}) = P(MS) \times P(\text{dress code})$

$\frac{15}{165} = \frac{45}{165} \times \frac{55}{165}$

Theorem 3

$P(MS \mid DC) = P(MS \mid DC')$

$\frac{15}{55} = \frac{30}{110}$

Therefore both theorems hold, and the events are independent.

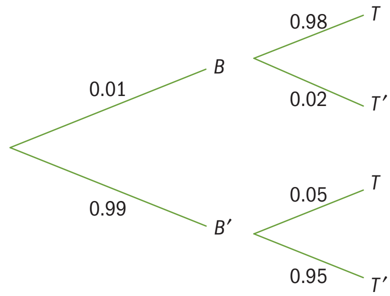
Review in context

1 a **i** $P(D) = \frac{45}{200} = \frac{9}{40}$ **ii** $P(N) = \frac{1}{2}$

iii $P(D \mid N) = \frac{1}{10}$ **iv** $P(D \mid N') = \frac{35}{100} = \frac{7}{20}$

b They are not independent from Theorem 3.

2 a



b $0.0098 + 0.9405 = 0.9503$

3 The events are independent.

4

	Smoker (*A*)	Non-smoker (*A'*)	
Cancer (*B*)	1763	981	2744
(*B'*)	73 237	124 019	197 256
	75 000	125 000	200 000

a $P(B \mid A) = \frac{1763}{75000}$

b $P(B \mid A') = \frac{981}{125\,000}$

c They are not independent from Theorem 2

d Students' own answer

5 a $P(B|A) = 0.69$ **b** $P(A) = 0.01$ **c** $P(B) = \frac{10}{330}$

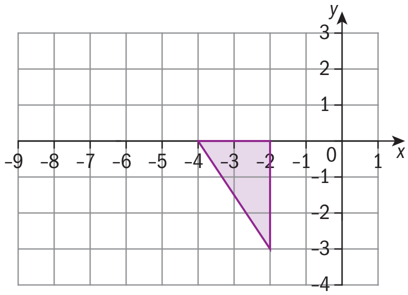
b The events are not independent.

Unit 5 Answers

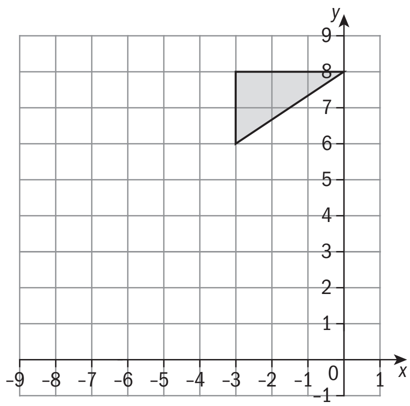
E5.1

You should already know how to:

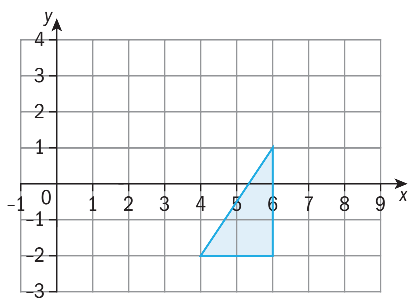
1 a



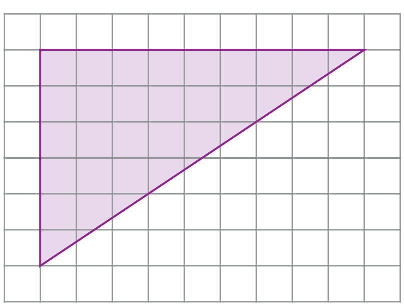
b



c



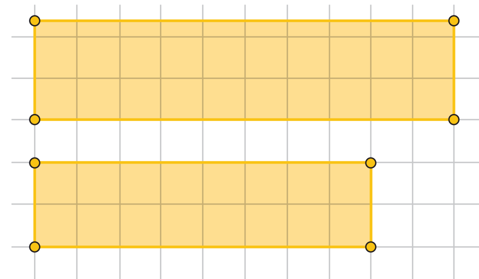
2



3 $y = 4x - 1$

Practice 1

1



2 a 5 **b** 2 **c** $\frac{1}{2}$ **d** $\frac{1}{4}$ **e** $\frac{3}{2}$

f $\frac{2}{3}$ **g** $\frac{5}{3}$ **h** $\frac{4}{5}$ **i** $\frac{5}{4}$ **j** $\frac{2}{5}$

3 a A + E
B + F + H
C + G
D is not similar to any other shape.

b From A to E: sf = 3; from E to A: sf = $\frac{1}{3}$

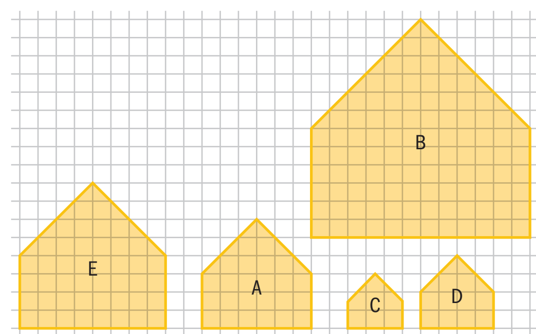
From B to F: sf = 2; from F to B: sf = $\frac{1}{2}$

From B to H: sf = 3; from H to B: sf = $\frac{1}{3}$

From F to H: sf = $\frac{3}{2}$; from H to F: sf = $\frac{2}{3}$

From C to G: sf = 2; from G to C: sf = $\frac{1}{2}$

4 a

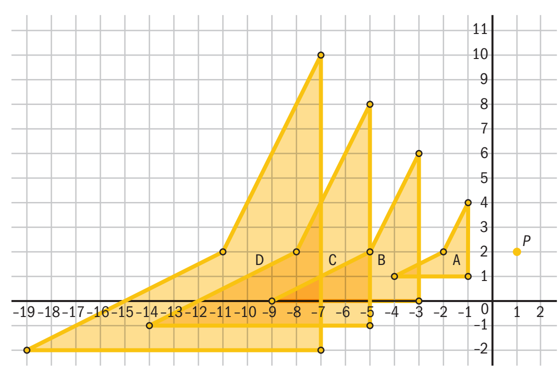


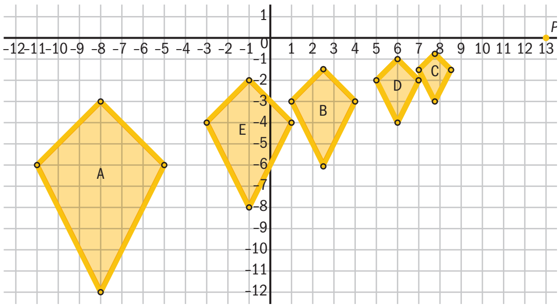
b Scale factor from C to D is $\frac{4}{3}$

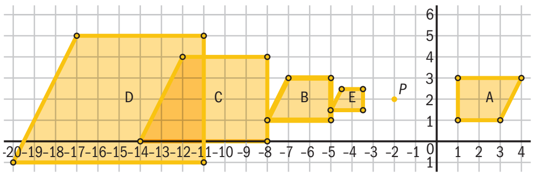
c Scale factor from E to C is $\frac{3}{8}$

Practice 2

1



2 

3 

4 B: dilation of scale factor $\frac{1}{2}$ around the center of dilation (0, 6)

C: dilation of scale factor $-\frac{1}{2}$ around the center of dilation (8, 2)

D: dilation of scale factor 2 around the center of dilation (8, 11)

E: dilation of scale factor −2 around the center of dilation (9, 5)

Practice 3

1 **a** No because $-6 \times 6 \neq -1$, so their gradients are not perpendicular.

b No because $\frac{1}{6} \times 6 \neq -1$, so their gradients are not perpendicular.

c Yes because $-\frac{1}{6} \times 6 = -1$, so their gradients are perpendicular.

d Yes because $-\frac{1}{6} \times 6 = -1$, so their gradients are perpendicular.

e Yes because $-\frac{1}{6} \times 6 = -1$, so their gradients are perpendicular.

f No because $\frac{1}{6} \times 6 \neq -1$, so their gradients are not perpendicular.

2 $y = -\frac{1}{3}x + 2$

3 $x - y + 2 = 0$

4 $4x - 3y - 1 = 0$

5 (3, 4)

6 $x = 4$

7 It is a rectangle because the vertices are all 90° angles since the gradients of adjacent lines are negative reciprocals of each other: $DE \perp EF$, $EF \perp FG$, $FG \perp GD$ and $GD \perp DE$ (minimum 3 of the 4 are needed for justification).

Practice 4

1 **a** This is a reduction with a negative scale factor, because the large object that is far away (the Sun) is "transformed" into the smaller image that is inside the box (the image of the Sun). Each line that goes through a point on the Sun and its corresponding point on the image in the box passes through the pinhole.

b The center of dilation is the pinhole on the side of the box.

c The scale factor is negative because the image is on the opposite side of the center of dilation (the pinhole) than the original object (the Sun).

d The distance between the Sun and the pinhole is 151.82 million km, and the distance between the pinhole and the image is 1 m.

151 820 000 km = 151 820 000 000 m.

Thus, the scale factor is $-\frac{1}{151\,820\,000\,000}$.

e The radius of the Sun is 696 340 km.

696 340 km = 696 340 000 m

$696\,340\,000\text{ m} \times \frac{1}{151\,820\,000\,000} = 0.00458662\text{ m} =$

0.458662 cm = 4.5866 mm

f The image of the Sun is reversed compared to the real Sun because the scale factor is negative. However, since the Sun is roughly a sphere with no patterns that are visible to the naked eye, it will not be noticeable that its image is upside down in the pinhole camera.

2 **a** The lens

b The scale factor is −5 since the photograph on the table is 5 times the distance from the lens as the negative, but on the other side of the lens than the negative (hence a negative scale factor).

c 24 mm × 5 = 120 mm = 12 cm, 36 mm × 5 = 180 mm = 18 cm. So the maximum size of the photograph is 12 cm × 18 cm for this enlarger.

d Even though the scale factor is negative, the image is enlarged, meaning that the scale factor is smaller than −1 (or its absolute value is larger than 1).

Mixed practice

1 B: enlargement by scale factor 3 from point (−1, −3)

C: reflection in the line $y = 5$

D: translation by $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$

E: rotation 90° counterclockwise around (1,11)

F: reflection in the line $y = x + 5$

G: rotation 180° around point (0, 7) or enlargement by scale factor −1 from point (0,7)

H: enlargement by scale factor −2 around point (−2, 4)

I: translation by $\begin{pmatrix} -7 \\ 10 \end{pmatrix}$

2 It is not a rectangle because at least one pair of adjacent sides are not perpendicular (none of them are).

3 Yes the diagonals of ABCD are perpendicular because AC ⊥ BD.

4 **a** The flashlight represents the center of enlargement. Since the distance between the shadow and the flashlight (21 m) is three times the distance between the person and the flashlight (7 m), the scale factor is 3. The scale factor is positive because the image (the shadow) is on the same side of the center of enlargement (the flashlight) as the original object (the person).

b The height of the shadow would be triple the height of the person, since the scale factor is 3. The height is thus 528 cm.

c The point on the floor where the person is standing is the center of dilation. Since the flashlight is 7 m from the person and the shadow is 14 m from the person on the opposite side, the height of the shadow (the dilated distance) below the floor is obtained by a dilation with scale factor −2 of the height of the flashlight above the ground (the original distance).

d 88 cm × 2 = 176 cm. So out of the total length of the shadow (528 cm), 176 cm are cut off by the floor, leaving the visible shadow above the floor to be 352 cm long.

Unit 6 Answers

E6.1

You should already know how to:

1 a 3.9 **b** 80.5

2 a In a statistical study, the population is the entire group of individuals we want information about.

b A sample is any subset of the population.

Practice 1

1 a $\mu = 5, \sigma = 2.19$. Mean is 5, but on average the spread of the data is ±2.19 on each side.

b $\mu = 16.54, \sigma = 3.07$.
This implies that the mean is 16.54, but on average the spread of the data is ±3.07 on each side.

c $\mu = 15$ cm, $\sigma = 2.05$ cm.
This implies that the mean is 15 cm, but on average the spread of the data is ±2.05 cm on each side.

d $\mu = 40.78$ kg, $\sigma = 4.75$ kg.
This implies that the mean is 40.78 kg, but on average the spread of the data is ±4.75 kg on each side.

2 a $\mu = 25.4$ minutes, $\sigma = 2.82$ minutes.

b This implies that the mean length of journey is 25.4 minutes, but on average the spread of the length of the journey is ±2.82 minutes.

3 a Motorway: $\mu = 20$ min, $\sigma = 4.60$ min
Country lanes: $\mu = 20$ min, $\sigma = 1.41$ min

b Student's own answer.

4 $\mu = 8, \sigma = 4.32$

5 a $\mu = 1.72, \sigma = 0.0618$ **b** $\mu = 1.74, \sigma = 0.0906$

6 a $\mu = 7, \sigma = 2.45$

b Student's own answer

c $\mu = 11, \sigma = 2.45$

d The mean is affected but the standard deviation is not.

e $x = 20, y = 9$ or $x = -7.5, y = 3.5$

Practice 2

1 a $\mu = 2.94, \sigma = 1.27$

b $\mu = 15.03, \sigma = 6.67$

c $\mu = 10.46, \sigma = 5.34$

2 a i 63 kg **ii** 9.45 kg

b 53.55 kg

3

n	$\sum fx$	$\sum f(x-\mu)^2$	μ	σ
10	50	20	5	$\sqrt{2}$
20	123	41.2	6.15	$\sqrt{2.06}$
39	197	80.34	5.05	$\sqrt{2.06}$
43	223	102.34	5.186	$\sqrt{2.38}$
23	115	39.56	5	$\sqrt{1.72}$

4

Length (cm)	Mid-value (x)	f	fx	$x-\mu$	$f(x-\mu)^2$
$0 < x \le 5$	2.5	3	7.5	−11.4	389.88
$5 < x \le 10$	7.5	4	30	−6.4	163.84
$10 < x \le 15$	12.5	6	75	−1.4	11.76
$15 < x \le 20$	17.5	7	122.5	3.6	90.72
$20 < x \le 25$	22.5	5	112.5	8.6	369.8
		25	$\sum fx =$ 347.5		$\sum f(x-\mu)^2 =$ 1026

The mean is 13.9 and the standard deviation is 6.41.

Practice 3

1 $\mu = 11$ kg, $\sigma = 1.5$, kg. Fully grown mute swans will on average weigh between 9.5 and 12.5 kg.

2 a $\sum x = 27$ m, $\sum x^2 = 48.628$

b $\mu = 1.811$ m, $\sigma = 0.060$ m

c There is not a dramatic change in the mean, however the standard deviation has changed.
68% of the team should lie within 6 cm of the mean.

3 You would expect the rainfall to be within the range of 1109–1419 mm, so:

a 1100 mm is exceptional

b 1400 mm is not exceptional

4 a 49.56 min **b** 48.48 min

c Either the raw actual data or $\sum x^2$

5 $\mu = 90.63, \sigma = 33.02$

Practice 4

1 $s_{n-1} = 4.36$ **2** $\bar{x} = 460.16$ $s_{n-1} = 18.73$

3 $\bar{x} = 1550$ hours, $s_{n-1} = 39.53$ hours

4 b The data is symmetrically distributed about the mean and appears to follow a normal distribution.

c $x = 7.84$ people, $s_{n-1} = 2.85$

d Katie could assume that 68% of carriages would have between 5 and 11 people in them.

Mixed practice

1 a $\mu = 4.4, \sigma = 1.36$ **b** $\mu = 24.5$ kg, $\sigma = 2.93$ kg

c $\mu = 4, \sigma = 0.76$ **d** $\mu = 13.3, \sigma = 6.06$

2 With water: $\mu = 13.6, \sigma = 2.2$
With food: $\mu = 15.7, \sigma = 2.15$
On average the strawberries given food gave a better yield, the mean was greater and the standard deviation was lower.

3 a $\mu = 55.2, \sigma = 5.96$

b On an average day the hamster will eat about 55.2 g of food. About 68% of the time, the hamster will eat between 49.2 and 61.2 g of food.

c This is 5 standard deviations less than normal, so it is an abnormal result, and he should be worried.

4 b $4.25 - x(1.5) = 2, x = 1.5$
$4.25 + x(1.5) = 6.5, x = 1.5$

5 $\mu = 28.15$, $\sigma = 3.84$

6 a $\bar{x} = 200.5$, $s_n = 2.66$ **b** $\bar{x} = 200.5$, $s_{n-1} = 2.80$

7 a $\bar{x} = 9.5$, $s_n = 2.49$
$\bar{x} = 9.5$, $s_{n-1} = 2.62$

b Yes, because the mean is close to 10 and assuming a normal distribution 50% of the values will be over 9.5 and 34% of the values will be between 9.5 and 12.12, so many will be over 10.

Review in context

1 Patient 1:
$\bar{x} = 91.3$, $s_{n-1} = 5.12$ high blood pressure treatment
Patient 2:
$\bar{x} = 78.3$, $s_{n-1} = 2.63$ no treatment
Patient 3:
$\bar{x} = 78.7$, $s_{n-1} = 15.0$ more tests needed

2 $\bar{x} = 75$, $s_{n-1} = 5.57$
Provided the data is normally distributed you can assume 68% lie between 70 and 80.

3 a $\bar{x} = 3.2$ kg, $s_n = 0.49$, $s_{n-1} = 0.490$

b $\bar{x} = 1.96$ kg

4 a $\bar{x} = 31.79$, $s_n = 6.49$, $s_{n-1} = 6.50$

b Provided the data is normally distributed you can assume that now 68% of the ages of first time mothers lie between 25 and 38 however 30 years ago 68% of the ages of first time mothers were between 22 and 28 years old.

Unit 7 Answers

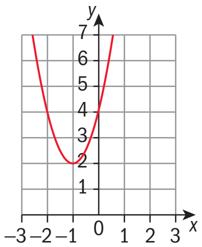
E7.1

You should already know how to:

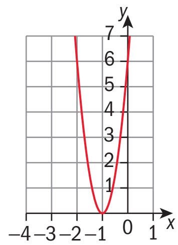
1 a $\frac{5}{13}$ **b** $\frac{12}{13}$

2 a 0.707 (3 s.f.) **b** 0.799 (3 s.f.)

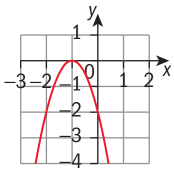
3 a



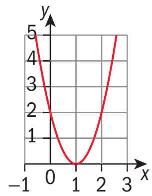
b



c



d



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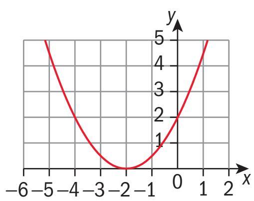
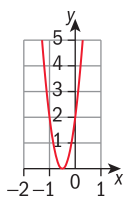
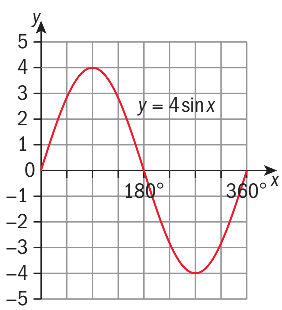


f

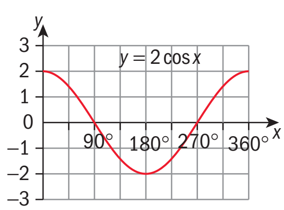


Practice 1

1 a



b

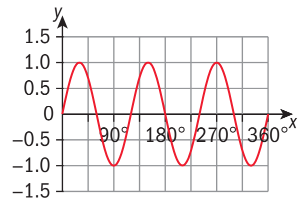


2 a $y = 3\sin x$ **b** $y = 5\cos x$
c $y = 0.25\cos x$ **d** $y = 1.5\sin x$

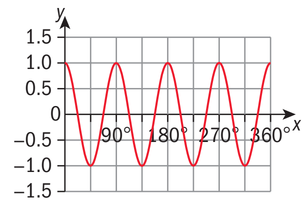
3 a $y = 2\cos x$
b $y = 0.5\sin x$ and $y = 2\sin x$
c $y = 2\cos x$ and $y = 4\cos x$
d $y = 0.5\sin x$

Practice 2

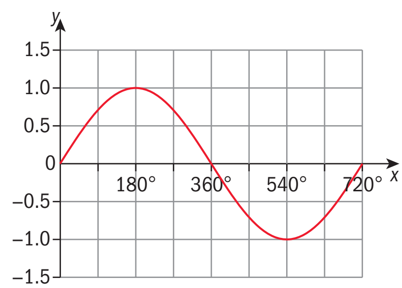
1 a Period 120°, Frequency 3



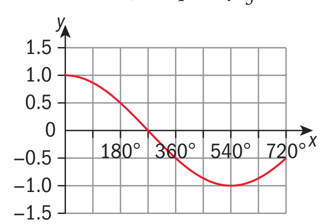
b Period 90°, Frequency 4



2 a Period 720°, Frequency $\frac{1}{2}$



b Period 1080°, Frequency $\frac{1}{3}$

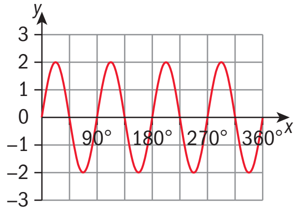


3 a $y = \sin\left(\frac{x}{3}\right)$ **b** $y = \cos(3x)$
c $y = \cos\left(\frac{x}{4}\right)$ **d** $y = \sin(4x)$

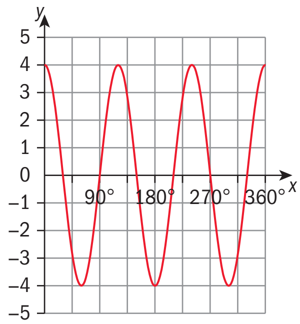
4 a $y = 3\sin\left(\frac{x}{2}\right)$ **b** $y = 0.5\cos(3x)$
c $y = 0.5\cos(0.5x)$ **d** $y = 3\sin(2x)$

Practice 3

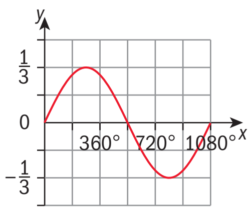
1 a Amplitude 2, Period 90°, Frequency 4



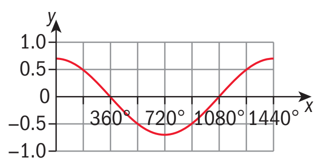
b Amplitude 4, Period 120°, Frequency 3



c Amplitude $\frac{1}{3}$, Period 1080°, Frequency $\frac{1}{3}$



d Amplitude 0.7, Period 1440°, Frequency $\frac{1}{4}$

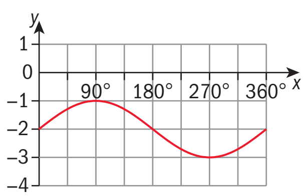


Practice 4

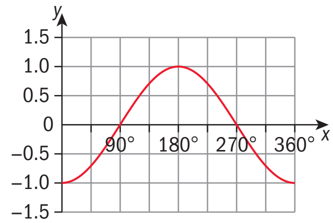
1 a Vertical translation 7 units in the positive direction
b Reflection in the x-axis
c Vertical translation 2 units in the negative direction
d Reflection in the y-axis

2 a $y = \sin x + 2$ **b** $y = -\sin x$ or $y = \sin(-x)$
c $y = -\cos x$ **d** $y = \sin x - 4$

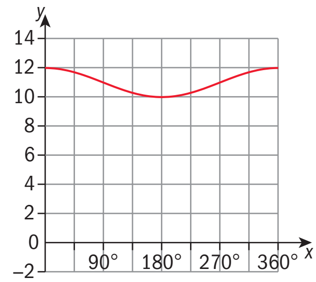
3 a Amplitude 1, Period 360°



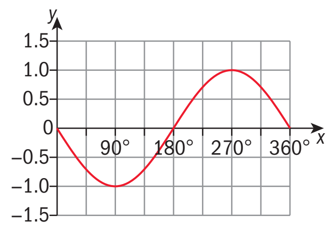
b Amplitude 1, Period 360°



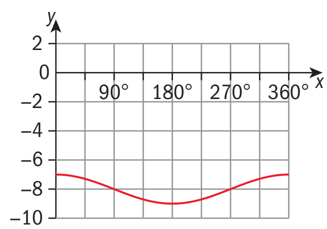
c Amplitude 1, Period 360°



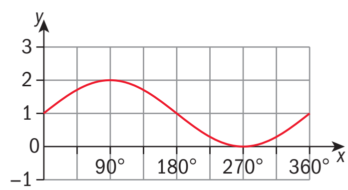
d Amplitude 1, Period 360°



e Amplitude 1, Period 360°



f Amplitude 1, Period 360°



4 a Amplitude 1, Period 360°, Frequency 1
b $y = \sin x + 4$
c Move down 3 units
d $y = \sin x + 1$

5 Both are correct – both transformations on the graph of the given function are equivalent.

Practice 5

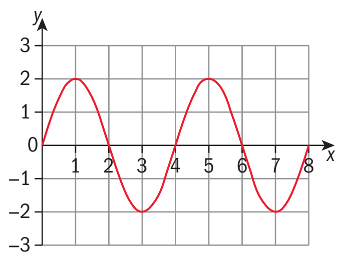
1 a Vertical dilation scale factor 4 and reflection in the x-axis
b Vertical dilation scale factor 2 and vertical translation 9 units in the negative direction
c Vertical dilation scale factor 0.5 and horizontal translation 4 units in the positive direction
d Horizontal dilation scale factor 2 and vertical translation 3 units in the positive direction
e Horizontal dilation scale factor 3 and reflection in the x-axis
f Horizontal dilation scale factor $\frac{1}{2}$ reflection in the y-axis and vertical translation 1 unit in the positive direction.

2 a Vertical dilation scale factor 3 and reflection in the x-axis
b Vertical dilation scale factor 2 and vertical translation 1 unit in the positive direction
c Horizontal dilation scale factor 2 and vertical translation 2 units in the positive direction
d Vertical dilation scale factor 0.5 and vertical translation 1 unit in the negative direction
e Horizontal dilation scale factor $\frac{1}{3}$ and reflection in the x-axis
f Horizontal dilation scale factor 2, reflection in the y-axis and vertical translation 3 units in the negative direction

3 a $y = 3\sin(2x)$ **b** $y = -3\cos\left(\frac{x}{2}\right)$
c $y = -\cos x + 3$ **d** $y = -3\sin\left(\frac{x}{2}\right) - 2$
e $y = 0.5\cos(2x) + 1$ **f** $y = -2\cos(3x) + 2$
g $y = -2\sin\left(\frac{x}{2}\right) + 1$ **h** $y = -1.5\cos x + 3$

Practice 6

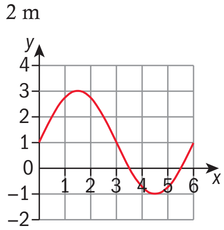
1 a i 1 liters/s **b** −1.64 liters/s (to 3 s.f.) **c** 0 liters/s
b Exhaling
c Period is 4 seconds – represents your breathing rate
d Amplitude = 2, Max velocity = 2 liters/s
e



2 $y = 0.5\sin(90°t) + 2$

3 a $y = 2.5\cos(2x) - 0.5$ **b** $y = 3\sin(18°x) + 1$
c $y = 2\cos(6x) - 3$ **d** $y = -3\sin 12°x + 4$

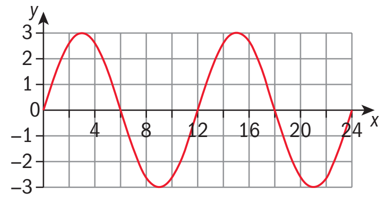
4 a 2 m
b



c No – max height is 3 m

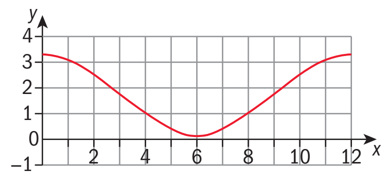
5 a 1 m **b** 7 m **c** 2 seconds
d 1 m **e** 8 seconds **f** 3 m
g $y = -3\cos(45°t) + 4$

6 a 0, 0, 0, 0, 0
b



c 3 am and 3 pm, both 3 meters
d 2.60 m
e From 1:24 to 4:36 (both am and pm)

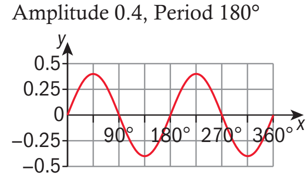
7 a $y = 1.6\cos(30°t) + 1.7$
b



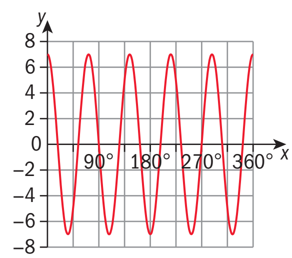
Mixed practice

1 a Amplitude 4, Period 360°, Frequency 1, $y = 4\cos x$
b Amplitude 2, Period 90°, Frequency 4, $y = 2\cos(4x)$
c Amplitude 0.5, Period 120°, Frequency 3, $y = 0.5\cos(3x)$

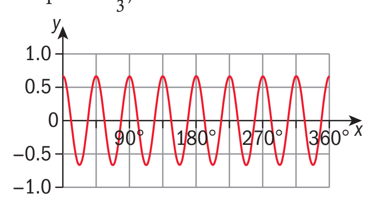
2 a Amplitude 0.4, Period 180°



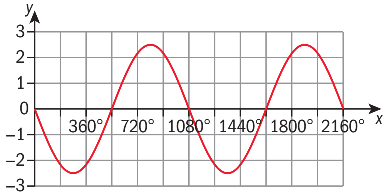
b Amplitude 7, Period 72°



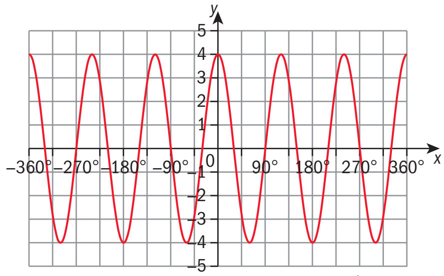
c Amplitude $\frac{2}{3}$, Period 45°



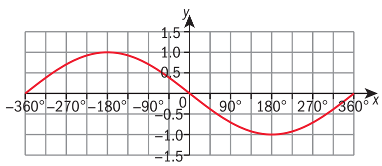
d Amplitude $\frac{5}{2}$, Period 1080°



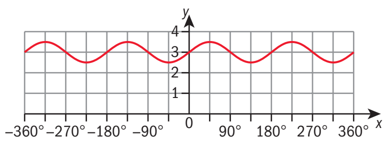
3 a Amplitude 4, Period 120°, Frequency 3



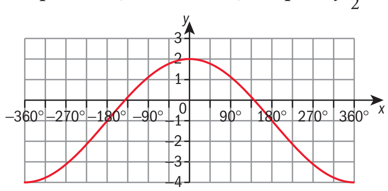
b Amplitude 1, Period 720°, Frequency $\frac{1}{2}$



c Amplitude 0.5, Period 180°, Frequency 2

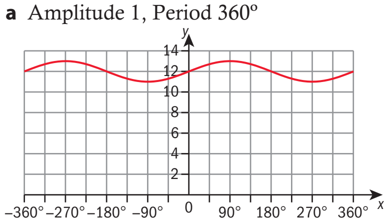


d Amplitude 3, Period 720°, Frequency $\frac{1}{2}$

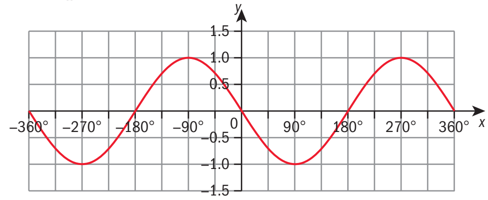


4 $y = -2\sin\left(\frac{x}{2}\right)$

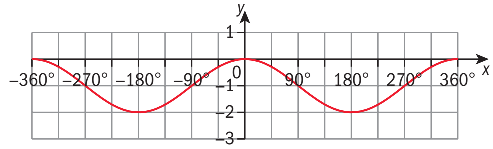
5 a Amplitude 1, Period 360°



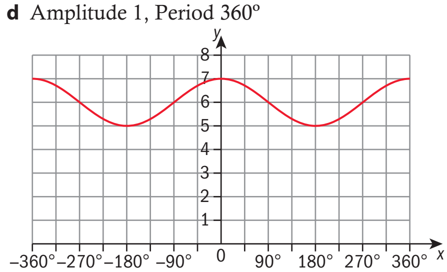
b Amplitude 1, Period 360°



c Amplitude 1, Period 360°



d Amplitude 1, Period 360°



6 a Amplitude 1, Period 360° $y = \sin x + 3$

b Amplitude 1, Period 360° $y = \cos x - 3$

c Amplitude 1, Period 360° $y = -\cos x$

d Amplitude 1, Period 360° $y = \sin x - 1$

7 a Vertical dilation scale factor 3 and vertical translation 2 units in the positive direction

b Vertical dilation scale factor 2, reflection in the x-axis, and horizontal dilation scale factor $\frac{1}{4}$

c Horizontal dilation scale factor $\frac{1}{2}$, reflection in the x-axis, vertical translation 3 units in the negative direction

d Horizontal dilation scale factor 3, vertical dilation scale factor $\frac{1}{2}$, and vertical translation 4 units in the negative direction

e Horizontal dilation scale factor $\frac{1}{3}$, and vertical dilation scale factor 3, reflection in the x-axis, and vertical translation 3 units in the positive direction

f Horizontal dilation scale factor $\frac{4}{3}$, vertical dilation scale factor $\frac{3}{4}$.

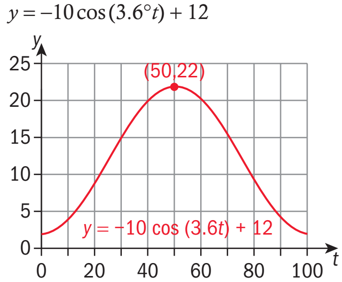
8 a $y = 2\sin x - 1$ **b** $y = -\cos(2x) - 2$

c $y = -\sin\left(\frac{x}{2}\right) + 1$ **d** $y = \sin(4x) + 1$

e $y = -2\cos(2x) - 2$

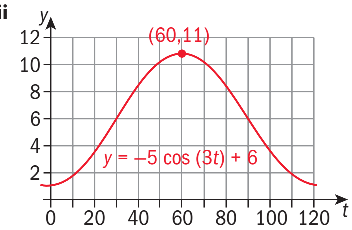
9 a i $y = -10\cos(3.6°t) + 12$

ii



b **i** $y = -5\cos(3°t) + 6$

ii



iii Children

10 **a** The period is 10 seconds – it is the time taken for the building to sway back to its original position.

b 20 cm

c $y = 20\sin(36x)$

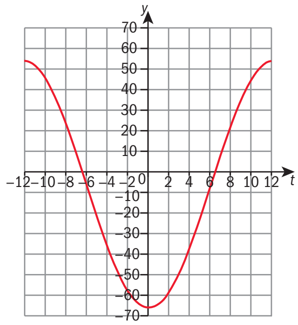
11 **a** 180 cm – represents the distance travelled in one revolution

b 28.65 cm; the amplitude is how far the nail travels above and below the wheel's axle.

c 28.65 cm

d $h = 28.65 - 28.65\sin(2x)$

12 **a**



b Sunrise and sunset

c The sun is below the horizon – it is dark.

d 37.4°, 47°

e 6:19 am

Review in context

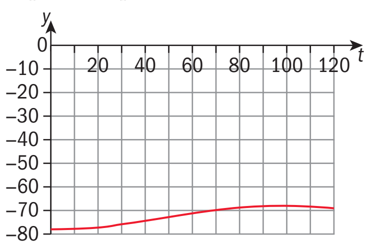
1 **a** $d = -10\sin\left(\frac{360x}{15}\right) + 9$

b **i** 4.9 m **ii** 0.40 m **iii** 17.7 m

c –1 m

d The shore waters would recede leaving the ocean floor exposed.

2 **a** Amplitude 5, period 200 hours



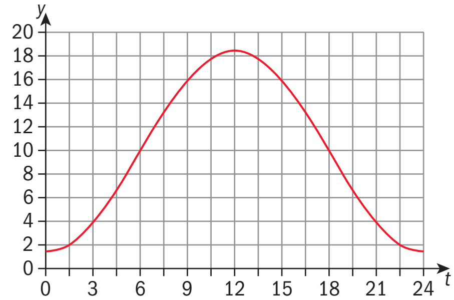
b $y = -5\cos\left(\frac{9}{5}x\right) - 73$

c –73°

d Horizontal dilation scale factor $\frac{5}{9}$, vertical dilation scale factor 5, reflection in the x-axis, and vertical translation 73 units in the negative direction

e No – it would travel linearly down the latitude

3 **a**



b $y = -8.5\cos(15°t) + 10$ **c** 4m, 16m

d Midnight and midday

e Tides provide constant energy, whereas wind varies.

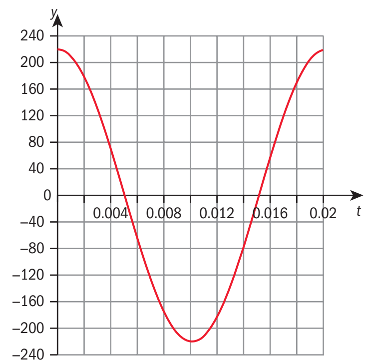
4 **a**

b Amplitude: 1.5 million, period 20 days, frequency 18.
$y = 1.5\cos(18x) + 3.5$

c More than half – is at 3.5 million (> 2.5 million)

d Before – the next low would happen sooner and be over quicker.

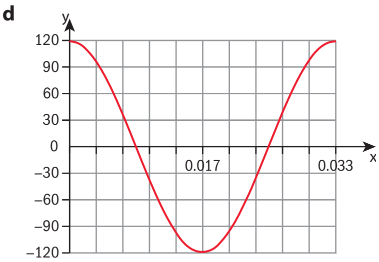
5 **a**



b Frequency = $\frac{1}{50}$ seconds/cycle, hence period is $\frac{360°}{\left(\frac{1}{50}\right)} = 18\,000°$

c $y = 220\cos(18\,000°t)$

d



e 21600, 120, $y = 120\cos(21\,600°t)$

f Vertical dilation scale factor $\frac{6}{11}$ and horizontal dilation scale factor $\frac{5}{6}$

E7.2

You should already know how to:

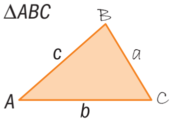
1 **a** $10.5\ cm^2$ **b** $42\ cm^2$

2 **a** 3.76 cm **b** 45.6°

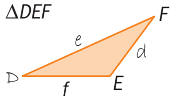
3 $36.1\ cm^2$

Practice 1

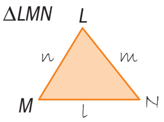
1 ΔABC



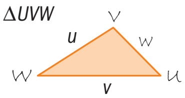
ΔDEF



ΔLMN



ΔUVW



Practice 2

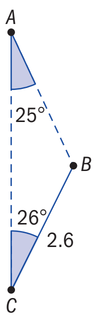
1 **a** $a = 22.9$ cm **b** $b = 16.1$ cm **c** $c = 6.14$ cm

2 **a** $A = 44°$, $B = 56°$ **b** $A = 82°$, $B = 28°$ **c** $B = 57°$, $C = 55°$

3 **a** $x = 59.5°$ **b** $x = 33.5°$ **c** $x = 39.3$ cm

4 $y = 12.4$ cm

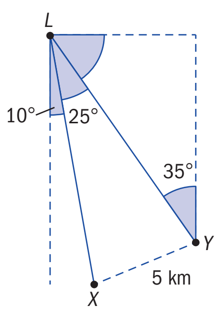
5 **a**



b $\angle CBA = 129°$, $\angle CAB = 25°$

c $c = 2.7$ km, distance = 5.3 km

6 **a**



b $L = 25°$, $X = 30°$, $Y = 125°$

c 5.92 km

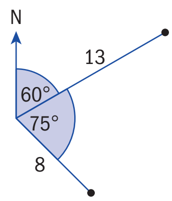
Practice 3

1 **a** $a = 6.3$ cm **b** $b = 6.0$ cm **c** $c = 61.1$ cm

2 **a** 91° **b** 103° **c** 42°

3 **a** $r = 19.0$ **b** $E = 104°$ **c** $y = 11.0$ **d** $D = 73.9°$

4 **a**



b 75° **c** 13.4 km

5 **a** 86.4° **b** 34.8°

Practice 4

1 **a**, **b**, **e**: the cosine rule; three sides and an angle are involved in the problem.
c, **d**, **f**: the sine rule; each problem involves two sides and two angles.

2 **a** 11.7 cm **b** 95.0 cm **c** 132°
d 74.6° **e** 112 cm **f** 26.8°

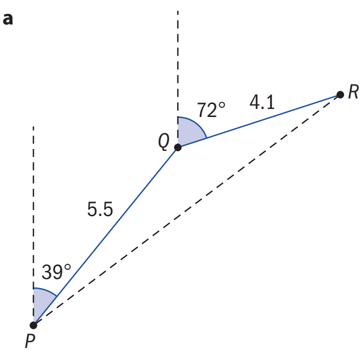
3 **a** $a = 21.3$ cm **b** $b = 8.27$ cm **c** $c = 24.1°$
d $d = 70.6°$ **e** $e = 104°$ **f** $f = 10.1$ cm

4 $x = 4$

5 **a** 7.65 cm **b** 79.2° **c** $11.1\ cm^2$

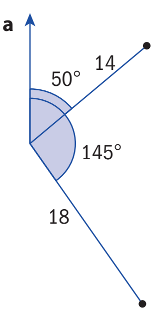
Practice 5

1 **a**



b 147° **c** 9.21 km

2 **a**



b 95° **c** 23.7 km

3 **c** 560 m

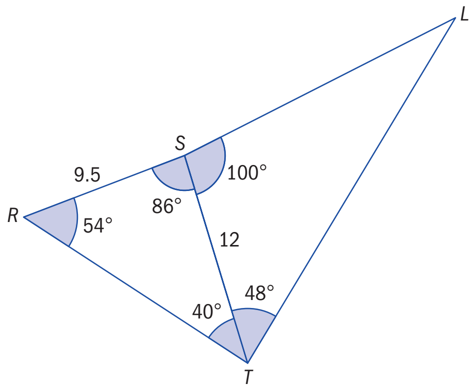
4 **a** 50° **b** 039° **c** 112 m **d** 211 m

Mixed practice

1 $a = 5.8$ cm, $b = 6.76$ cm, $c = 14.1$ cm, $d = 11.2$ cm, $e = 10.5$ cm, $f = 14.7$ cm
2 $a = 43.6°$, $b = 38.7°$, $c = 54.8°$, $d = 101°$
3 $a = 98.2°$, $b = 31.4°$, $c = 20.4°$, $d = 20.8°$
4 20.0°
5 42.6 m

Review in context

1 b $\angle RST = 86°$, $\angle STR = 40°$, $\angle TRS = 54°$
c $RT = 14.7$ km, $ST = 12.0$ km
d



e $LS = 16.8$ km, $LT = 22.2$ km

2 a 146° **b** 18.1 km
3 a 77.6° **b** 122° **c** 241°
4 8.79 km

Unit 8 Answers

E8.1

You should already know how to:

1 a 5, 6, 7 **b** 1, 3, 5 **c** 2, 4, 8

2 $u_{n+2} = 2u_{n+1} - u_n + 2$, $u_1 = 1$, $u_2 = 4$; $u_n = n^2$

3 a 3^7 **b** 6^2 **c** 2×5^n **d** 4^5

4 $x = 3$, $y = 9$

5 $173.64

Practice 1

1 a $u_n = 7n - 2$ **b** $u_n = 3n + 1$

c $u_n = 10 \times 4^{n-1}$ **d** $u_n = 3 \times \left(\frac{1}{3}\right)^{n-1}$

e $u_n = 20 \times \left(\frac{1}{4}\right)^{n-1}$

2 a 3, 5 **b** −2, 2 **c** 11, −4 **d** −13, −5

3 a 4, 5 **b** $\frac{1}{2}$, 7 **c** 15, 5

d $\frac{8}{3}$, $\frac{1}{3}$ **e** 4000, 10 **f** 7, $\frac{1}{2}$

4 $l = 6.4 - 0.08n$; 4 km

5 a $t = 60 \times 2^n$ **b** 7 864 320

Practice 2

1 31

2 43

3 16, −6.5, −2.5

4 −3, −9

5 $\frac{1}{3}$, 27th

6 3, 648

7 2, 192

8 20, $u_n = 20 \times 1.5^{n-1}$, 11th

9 1st term = 4
1st sequence common difference = 3
2nd sequence common difference = 4

Practice 3

1 $35 516.96

2 a Constant distance between rungs
b 20 cm, 15 cm
c 305 cm
d 23

3 a 12.5 cm
b 8.14 hours (3 s.f.)

4 a Each infected computer infects 5 more per day, meaning 6 per day are infected for each infected computer. These six then infect another 5 each meaning the total number of infected computers is 6 times more each day.
b 2
c On the 9th day

5 81 seats

6 a Deaths per month is double the previous month.
b 63; $d = 63 \times 2^n$
c 2016
d e.g. The infection can double in size only for a short period of time because soon the population size will be exceeded.

Mixed practice

1 a 18 **b** $u_{n+1} = u_n + 18$, $u_1 = 7$ **c** $u_n = 18n - 11$
d 259 **e** 21st

2 a 38 **b** 139 **c** 531

3 a $31 = u_1 + 2d$; $52 = u_1 + 5d$
b 17, 7, 80

4 −13

5 a 30, 15 **b** No

6 a 6, 7 **b** $u_{n+1} = 7u_n$, $u_1 = 6$ **c** $u_n = 6 \times 7^{n-1}$
d 14 406 **e** 4 941 258 (8th term)

7 a 6, 0.5 **b** $u_{n+1} = 0.5u_n$, $u_1 = 6$
c $u_n = 6 \times 0.5^{n-1}$ **d** 0.0117

8 a $4u_1$, $u_1 + 9$ **b** $4u_1 = u_1 + 9$, $u_1 = 3$ **c** 5

9 a $r^2 = \frac{48}{12}$ hence $r^2 = 4$ **b** 2, −2 **c** 384, −384

Review in context

1 a 4, 6, 8 – sequence has a common difference
b $u_n = 2n + 2$
c 42
d 70

2 a Same scale factor means common ratio
b 232 831 (nearest person)
c 3 388 132 (nearest person)

3 $183, $58

4 a Constant number of components per hour = common difference
b 2400
c 31 200

E8.2

You should already know how to:

1 a $\frac{1}{5}$ **b** $\frac{3}{8}$ **c** $\frac{y}{x^3}$ **d** $\frac{8}{3x}$

2 a $(x + 3)(x + 2)$ **b** $(x - 5)(x + 2)$ **c** $(x - 3)(x + 3)$

Practice 1

1 $5a^2$; $a \neq 0$ **2** $\frac{5x+3}{2}$; $x \neq 0$

3 $\frac{6x^3}{5y^2}$; $x, y \neq 0$ **4** $\frac{x-7}{x+7}$; $x \notin \{-8, -7\}$

5 $\frac{a+2b}{2b-a}$; $a \notin \{b, 2b\}$ **6** $\frac{x+4}{x+5}$; $x \notin \{-5, -2\}$

7 $\frac{x+4}{x+2}$; $x \notin \{-2, 3\}$ **8** $\frac{x+2}{x+7}$; $x \notin \{-7, 5\}$

9 $\frac{x(x-8)}{x-3}$; $x \notin \{3, 4\}$

10 Does not simplify; $x \notin \{-3, 6\}$

11 Any unsimplified expression where the numerator is a multiple of either $(x + 2)$ or $(x - 5)$ or both, and where the denominator is a multiple of $(x + 2)(x - 5)$.

12 $\frac{x^2-3x-18}{x^2-16x+60}$ or an equivalent algebraic fraction where the numerator and denominator have both been multiplied by the same factor(s).

Practice 2

1 $\frac{6p^2}{k^2}$; $k \neq 0$

2 $\frac{c}{4p^2}$; $c, p \neq 0$

3 $\frac{5}{2}$; $x \notin \{-2, 3\}$

4 $\frac{2(x-2y)(2x+9y)}{x+3y}$; $x \notin \{-\frac{9}{2}y, -3y\}$

5 $\frac{x-2}{x-1}$; $x \notin \{-1, 1\}$

6 $\frac{a-2b}{a}$; $a \notin \{0, 2b\}$

7 $\frac{x-5}{x-2}$; $x \notin \{-3, -2, -1, 2\}$

8 $\frac{(x-3)(x+6)}{(x-6)^2}$; $x \notin \{-8, -6, 3, 6\}$

9 $\frac{2}{3}$; $x \notin \{-3, -2, 0, 1, 3\}$

10 $\frac{6(x+2)^2}{(x-1)^2(x-2)}$; $x \notin \{-2, -1, 1, 2, 5\}$

Practice 3

1 $\frac{3y+4x}{xy}$; $x, y \neq 0$

2 $\frac{5}{4x}$; $x \neq 0$

3 $\frac{2x+1}{6x^2}$; $x \neq 0$

4 $\frac{2c+3b-5a}{abc}$; $a, b, c \neq 0$

5 $\frac{2x}{x^2-4}$; $x \notin \{-2, 2\}$

6 $\frac{(x-10)(x+2)}{(x+4)(x-3)} = \frac{x^2-8x-20}{x^2+x-12}$; $x \notin \{-4, 3\}$

7 $\frac{2x^2+3x+16}{(x+4)(x-2)}$; $x \notin \{-4, 2\}$

8 $\frac{x+1}{x-3}$; $x \neq 3$

9 $\frac{-(x+2)}{x^2+3x-10}$; $x \notin \{-5, 2\}$

10 $\frac{5x^2+15x+4}{(x-8)(x+3)} = \frac{5x^2+15x+4}{x^2-5x-24}$; $x \notin \{-3, 8\}$

11 $\frac{8x^2+12x-6}{x^3-x}$; $x \notin \{-1, 0, 1\}$

12 $\frac{-x(x^2-2x-47)}{4(x+3)(x-5)} = \frac{-x^3+2x^2+47x}{4x^2-8x-60}$; $x \notin \{-3, 5\}$

Mixed practice

1 $\frac{x^3}{3}$; $x, y \neq 0$

2 $\frac{5x^4-6x^2}{4} = \frac{x^2(5x^2-6)}{4}$; $x \neq 0$

3 $\frac{x-7}{x-11}$; $x \notin \{-3, 11\}$

4 $\frac{1}{x-4}$; $x \notin \{-4, 4\}$

5 $\frac{10+xy}{5x}$; $x \neq 0$

6 $\frac{6x^2y}{z^2}$; $x, y, z \neq 0$

7 $\frac{x^3y^3}{x-y}$; $x \notin \{-y, 0, y\}$, $y \neq 0$

8 $\frac{x^2-10xy-4}{4x}$; $x \neq 0$

9 $\frac{12x}{(x+5)(x-7)}$; $x \notin \{-5, 7\}$

10 $\frac{6-5x}{(x+9)(x-8)} = \frac{-5x+6}{x^2+x-72}$; $x \notin \{-9, 8\}$

11 $\frac{10x}{(x-1)(x+4)} = \frac{10x}{x^2+3x-4}$; $x \notin \{-4, 1\}$

12 $\frac{13}{x^2-1} = \frac{13}{(x+1)(x-1)}$; $x \notin \{-1, 1\}$

13 $\frac{12b+20a-ab(a+b)}{4ab}$; $a, b \neq 0$

14 $\frac{x^3-7x^2+10x-24}{6x(x-2)}$; $x \notin \{0, 2\}$

Review in context

1 a $\frac{1}{R_{tot}} = \frac{1}{x} + \frac{1}{x+4} + \frac{1}{2x+2} = \frac{5x^2+16x+8}{x(x+4)(2x+2)}$

hence $R_{tot} = \frac{(x^3+6x^2+8x)}{(5x^2+16x+8)}$

b $\frac{(x^3+6x^2+8x)}{(5x^2+16x+8)} + 2x + 2 = \frac{(11x^3+48x^2+56x+16)}{(5x^2+16x+8)}$

2 a $I = \frac{V}{R}$

b $r_{tot} = \frac{(5+3r)(6+r)}{(11+4r)}$; $I = \frac{12(11+4r)}{(5+3r)(6+r)}$

3 a 4

b Decreased 8 times

c Increased 96 times

d It needs to double

E8.3

You should already know how to:

1 a Domain is $\mathbb{R}$, range is $\mathbb{R}$. b Domain is $\mathbb{R}$, range is $\mathbb{R}^+$.

c Domain is $\mathbb{R}$, $x \neq 1$, range is $\mathbb{R}$, $y \neq 0$.

2 a $y \propto x$, so $y = kx$

b $y \propto \frac{1}{x}$, so $y = \frac{k}{x}$

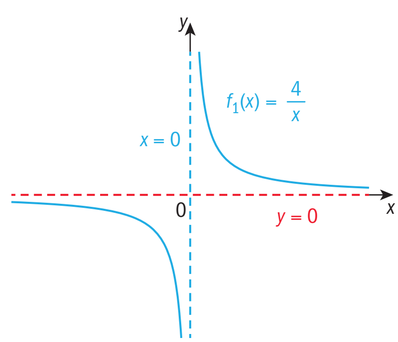
3 Horizontal translation 3 units in the negative x-direction, reflection in x-axis, vertical dilation scale factor 2, vertical translation 5 units in the positive y-direction.

4 a $y = \frac{4(x-2)}{3}$, which is a function

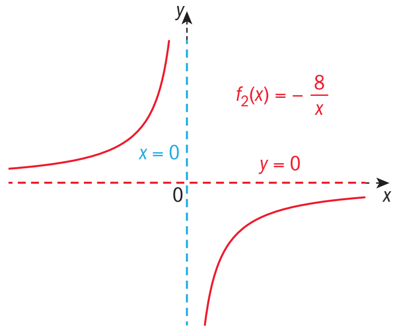
b $f^{-1}(x) = 2 \pm \sqrt{x}$, which is not a function

Practice 1

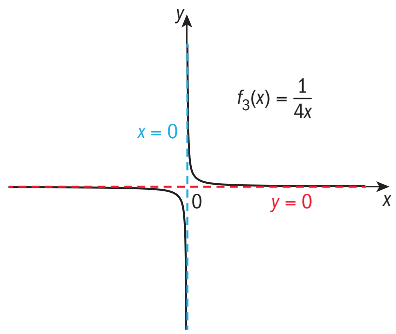
1 a Domain: $x \neq 0, x \in \mathbb{R}$, range: $y \neq 0, y \in \mathbb{R}$



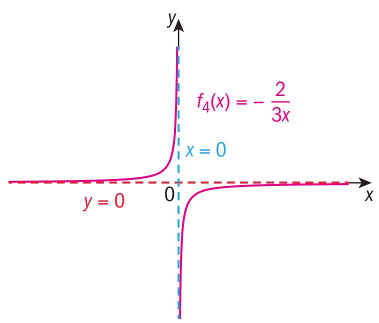
b Domain: $x \neq 0, x \in \mathbb{R}$, range: $y \neq 0, y \in \mathbb{R}$



c Domain: $x \neq 0, x \in \mathbb{R}$, range: $y \neq 0, y \in \mathbb{R}$



d Domain: $x \neq 0, x \in \mathbb{R}$, range: $y \neq 0, y \in \mathbb{R}$



2 b $y = 200$ hours

3 a The resistance is inversely proportional to the current.

b $k = 20$

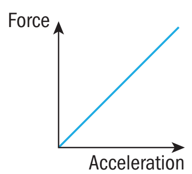
c $\mathrm{R}(\mathrm{I}) = \frac{20}{\mathrm{I}}$ or $\mathrm{R}(x) = \frac{20}{x}$

4 a 50 hours **b** 10 hours **c** 5 hours

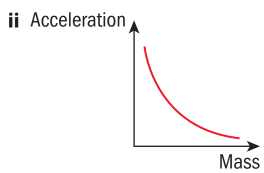
5 a i Force and acceleration are directly proportional.

ii Acceleration and mass are inversely proportional.

b i



ii



Practice 2

1 a $h = -3$, $k = -7$, asymptotes: $x = -3$, $y = -7$.
Domain: $x \neq -3, x \in \mathbb{R}$, Range: $y \neq -7, y \in \mathbb{R}$.

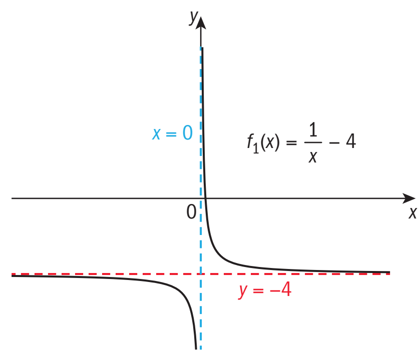
b $h = -2$, $k = 0$, asymptotes: $x = -2$, $y = 0$.
Domain: $x \neq -2, x \in \mathbb{R}$, Range: $y \neq 0, y \in \mathbb{R}$.

c $h = 5$, $k = 8$, asymptotes: $x = 5$, $y = 8$.
Domain: $x \neq 5, x \in \mathbb{R}$, Range: $y \neq 8, y \in \mathbb{R}$.

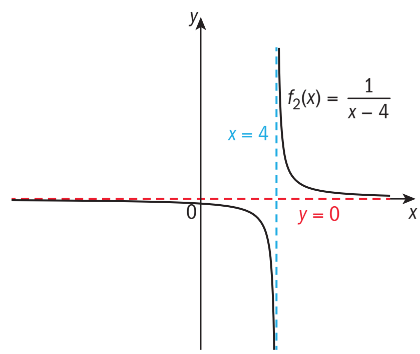
d $h = 0$, $k = -11$, asymptotes: $x = 0$, $y = -11$.
Domain: $x \neq 0, x \in \mathbb{R}$, Range: $y \neq -11, y \in \mathbb{R}$.

e $h = 1$, $k = \pi$, asymptotes: $x = 1$, $y = \pi$.
Domain: $x \neq 1, x \in \mathbb{R}$, Range: $y \neq \pi, y \in \mathbb{R}$.

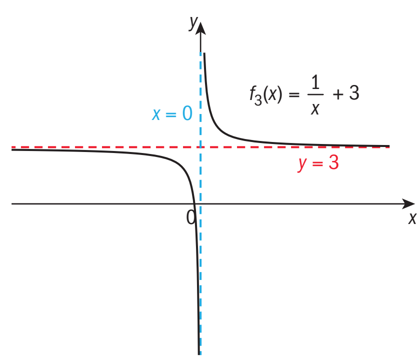
2 a Asymptotes: $x = 0$ and $y = -4$.
Domain: $x \neq 0, x \in \mathbb{R}$, Range: $y \neq -4, y \in \mathbb{R}$.



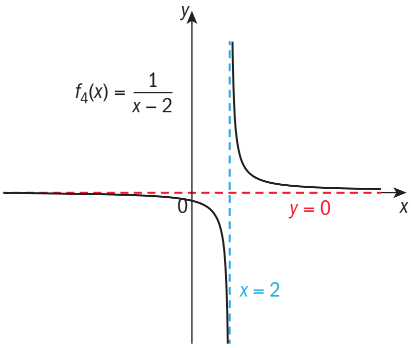
b Asymptotes: $x = 4$ and $y = 0$, Domain: $x \neq 4, x \in \mathbb{R}$,
Range: $y \neq 0, y \in \mathbb{R}$.



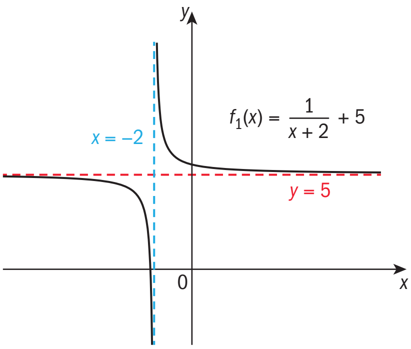
c Asymptotes: $x = 0$ and $y = 3$, Domain: $x \neq 0, x \in \mathbb{R}$,
Range: $y \neq 3, y \in \mathbb{R}$.



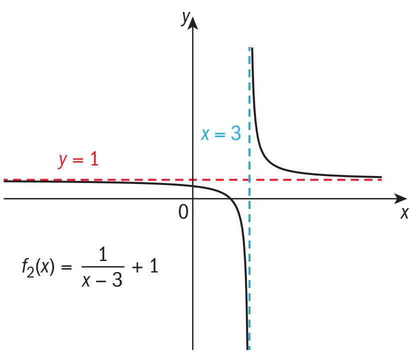
d Asymptotes: $x = 2$ and $y = 0$, Domain: $x \neq 2, x \in \mathbb{R}$, Range: $y \neq 0, y \in \mathbb{R}$.



e Asymptotes: $x = -2$ and $y = 5$, Domain: $x \neq -2, x \in \mathbb{R}$, Range: $y \neq 5, y \in \mathbb{R}$.



f Asymptotes: $x = 3$ and $y = 1$, Domain: $x \neq 3, x \in \mathbb{R}$, Range: $y \neq 1, y \in \mathbb{R}$.



3 a $y = \frac{1}{x} - 5$ **b** $y = \frac{1}{x-2} - 7$

c $y = \frac{1}{x+4}$ **d** $= \frac{1}{x-3} + 2$

4 a $y = \frac{1}{x-3} + 4$ **b** $y = \frac{1}{x} + 2$

c $y = \frac{1}{x-2} - 1$ **d** $y = \frac{1}{x+\frac{3}{4}} + \frac{4}{3}$

5 a i $h > 0$ horizontal translation in the positive x-direction

ii $h < 0$ horizontal translation in the negative x-direction

b i $k > 0$ vertical translation in the positive y-direction

ii $k < 0$ vertical translation in the negative y-direction

Practice 3

1 a $f(x) = \frac{1}{2(x-4)} - 1 = \frac{1}{2x-8} - 1$

b $f(x) = \frac{1}{\frac{1}{3}(x-3)} - 2 = \frac{3}{x-3} - 2$

c $f(x) = \frac{-\frac{1}{4}}{x} + 8 = -\frac{1}{4x} + 8$

d $f(x) = \frac{\frac{3}{2}}{-(x+2)} + 4 = \frac{-3}{2x+4} + 4$

2 a Vertical dilation of scale factor 3 (or horizontal dilation of scale factor $\frac{1}{3}$), horizontal translation of 6 units in the negative x-direction, vertical translation of 5 units in the negative y-direction. Asymptotes: $y = -5$, $x = -6$

b Horizontal translation of 2 units in the positive x-direction, vertical dilation of scale factor 5, vertical translation of 3 units in the positive y-direction. Asymptotes: $y = 3$, $x = 2$

c Horizontal translation of $\frac{1}{3}$ units in the negative x-direction, vertical dilation of scale factor 3. Asymptotes: $y = 0$, $x = -\frac{1}{3}$

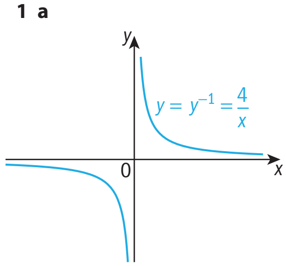
d Horizontal translation of 4 units in the positive x-direction, reflection in the x-axis, vertical dilation of scale factor $\frac{5}{2}$, vertical translation of 2 units in the positive y-direction. Asymptotes: $y = 2$, $x = 4$

e Horizontal translation of $\frac{1}{2}$ unit in the positive x-direction, reflection in the x-axis, vertical dilation of scale factor $\frac{5}{2}$ vertical translation of 8 units in the negative y-direction. Asymptotes: $y = -8$, $x = \frac{1}{2}$

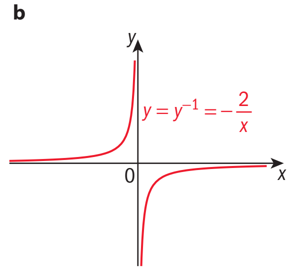
f Reflection in the y-axis (or in the x-axis), vertical dilation of scale factor 13 (or horizontal dilation of scale factor $\frac{1}{13}$), vertical translation of 11 units in the negative y-direction. Asymptotes: $y = -11$, $x = 0$

Practice 4

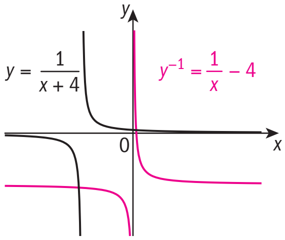
1 a



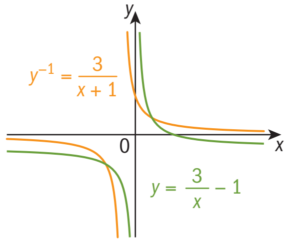
b



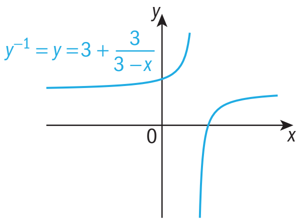
c



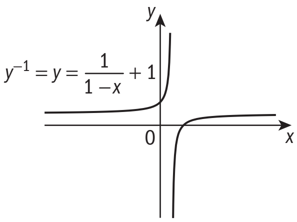
d



e



f



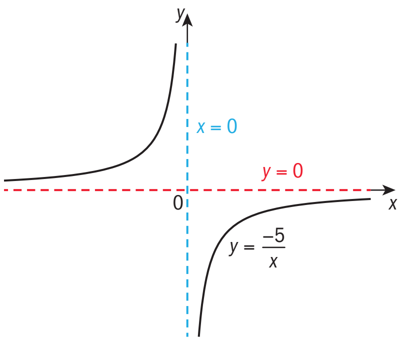
2 a $y = -\frac{5}{x} - 1$ **b** $y = \frac{7}{x-6} + 9$

c $y = -\frac{1}{x+8} + 12$ **d** $y = -\frac{6}{x+3} + 5$

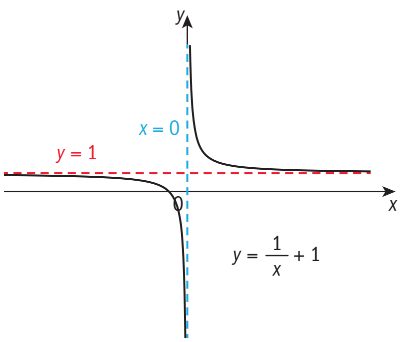
e $y = -\frac{10}{x-3} - 3 = \frac{3x+1}{3-x}$ **f** $y = -\frac{8}{x+2} + 4 = \frac{4x}{x+2}$

Mixed practice

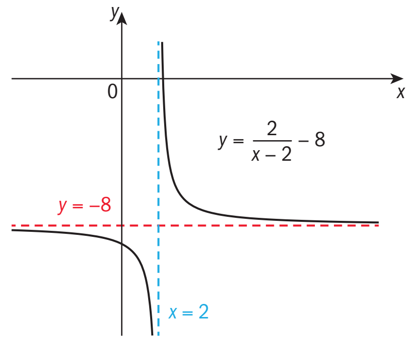
1 a Domain: $x \neq 0, x \in \mathbb{R}$, range: $y \neq 0, y \in \mathbb{R}$, asymptotes: $x = 0, y = 0$.



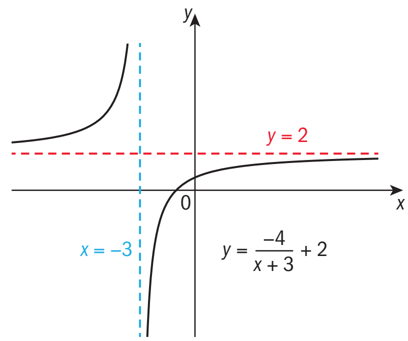
b Domain: $x \neq 0, x \in \mathbb{R}$, range: $y \neq 1, y \in \mathbb{R}$, asymptotes: $x = 0, y = 1$.



c Domain: $x \neq 2, x \in \mathbb{R}$, range: $y \neq -8, y \in \mathbb{R}$, asymptotes: $x = 2, y = -8$.



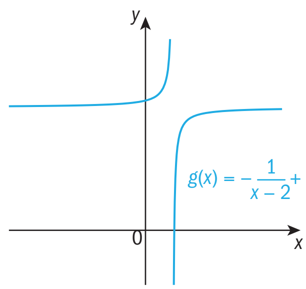
d Domain: $x \neq -3, x \in \mathbb{R}$, range: $y \neq 2, y \in \mathbb{R}$, asymptotes: $x = -3, y = 2$.



2 a $g(x) = \frac{3}{x-5} - 5$ **b** $g(x) = -\frac{1}{2x} + 1$

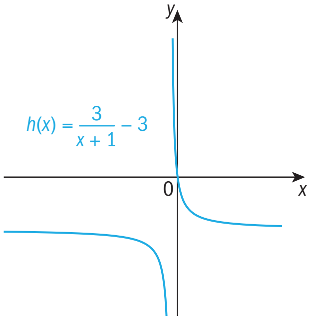
c $g(x) = \frac{-1}{x-3} - 2$ **d** $g(x) = -\frac{12}{x-4} - 1$

3 a Horizontal translation of 2 units in the positive *x*-direction, reflection in the *x*-axis vertical translation of 9 units in the positive *y*-direction. Asymptotes: $x = 2$ and $y = 9$.



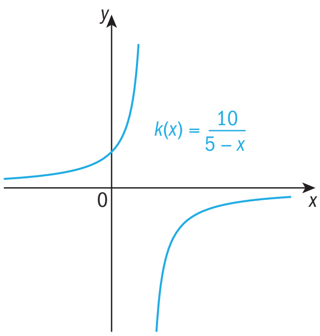
Inverse: $g^{-1}(x) = \frac{2x-19}{x-9}$

b Horizontal translation of 1 unit in the negative *x*-direction, vertical dilation of scale factor 3 vertical translation of 3 units in the negative *y*-direction.
Asymptotes: $x = -1$ and $y = -3$.



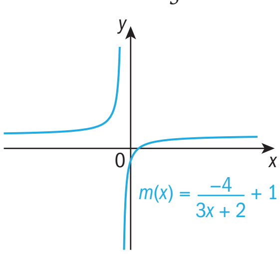
Inverse: $h^{-1}(x) = \dfrac{3}{x+3} - 1$

c Horizontal translation of 5 units in the positive x-direction, reflection in the x-axis, vertical dilation of scale factor 10
Asymptotes: $x = 5$ and $y = 0$.



Inverse: $k^{-1}(x) = -\dfrac{10}{x} + 5$

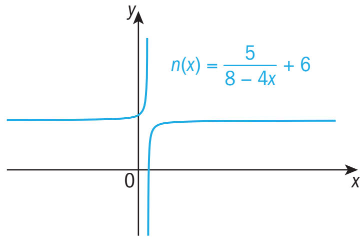
d Horizontal dilation of scale factor $\dfrac{1}{3}$, horizontal translation of $\dfrac{2}{3}$ units in the negative x-direction, reflection in the x-axis, vertical dilation of scale factor 4, vertical translation of 1 unit in the positive y-direction.
OR
Horizontal translation of $\dfrac{2}{3}$ units in the negative x-direction, reflection in the x-axis, vertical dilation of scale factor $\dfrac{4}{3}$ and vertical translation of 1 unit in the positive y-direction.
Asymptotes: $x = -\dfrac{2}{3}$ and $y = 1$.



Inverse: $m^{-1}(x) = \dfrac{-4}{3x-3} - \dfrac{2}{3}$

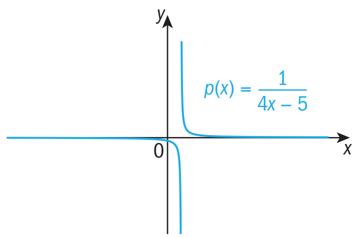
e Horizontal translation of 2 units in the positive x-direction, reflection in the x-axis, vertical dilation of scale factor $\dfrac{5}{4}$ vertical translation 6 units in the positive y-direction.

Asymptotes: $x = 2$ and $y = 6$.



Inverse: $n^{-1}(x) = \dfrac{-5}{4x-24} + 2$

f Horizontal translation of $\dfrac{5}{4}$ units in the positive direction, vertical dilation of scale factor $\dfrac{1}{4}$ (or horizontal dilation of scale factor 4).
Asymptotes: $x = \dfrac{5}{4}$ and $y = 0$



Inverse: $p^{-1}(x) = \dfrac{1}{4x} + \dfrac{5}{4}$

4 a 22.7 mg (3 s.f.) **b** $C = -\dfrac{600}{x+12} + 50$

c Horizontal translation of 12 units in the negative x-direction, reflection in the x-axis, vertical dilation of scale factor 600, vertical translation of 50 units in the positive y-direction.

Review in context

1 854.39 AUD

2 a slow pipe $= \dfrac{1}{3p}$ **b** $\dfrac{1}{p} + \dfrac{1}{3p}$

c $\dfrac{1}{p} + \dfrac{1}{3p} = \dfrac{1}{14}$, $p = \dfrac{56}{3} = 18.7$ hours.

Time to fill with slow pipe = 56 hours.

E8.4

You should already know how to:

1 a 4 **b** 5.5 **c** $\dfrac{20}{3}$

2 a $\dfrac{13x}{12}$ **b** $\dfrac{2(x-5)}{x(x+2)}$ **c** $\dfrac{2}{5}$ **d** $\dfrac{4x(x-7)(x+7)}{(x-3)(x-14)}$

3 a $(x-4)(x+4)$ **b** $3x(x-2)$
c $(x-6)(x+1)$ **d** $(3x+4)(x-3)$

4 a ± 3 **b** $-3, 1$ **c** $-3, -\dfrac{1}{2}$ **d** $\dfrac{1}{3}, 1$

Practice 1

1 $\frac{40}{17}$

2 $\frac{129}{47}$

3 $-\frac{14}{9}$

4 $\frac{15}{58}$

5 $-\frac{19}{8}$

6 0

7 $\frac{73}{29}$

8 $\frac{3}{26}$

9 1.43, −1.87 (3 s.f.)

Practice 2

1 **a** x **b** $6x$ **c** $3x^4$
d $x(x-2)$ **e** $2x$ **f** $(x-2)(x+2)$
g $x(x-2)(x+2)$ **h** $x(x^2+1)$ **i** $(x-1)(x+1)$
j $x(x-1)(x+1)$ **k** $2x+8$ **l** $(x+1)(x-1)$
m $(x-a)(x+1)(x-1)$ **n** $x(x-3)(x+2)$ **o** $2x(2x-3)(x+1)$
p $6x^2(x+2)(x-2)$

Practice 3

1 $-\frac{8}{5}$ 2 $\frac{4}{5}$ 3 $\frac{4}{5}$ 4 $\frac{4}{3}$

5 No solutions, $x = 2$ is an extraneous solution

6 $\frac{9}{7}$

7 No solutions,

8 8, −9

9 3.303, −0.303 (3 d.p.)

Practice 4

1 1

2 −1

3 No solutions

4 2

5 −6

Practice 5

1 No solutions, $x = 2$ is an extraneous solution

2 8

3 1, 1.5

4 $\frac{8}{5}$

5 ±4

Practice 6

1 **a** 30 **b** 20 cents (A is cheaper)

2 **a** 4 hours
b The other solution was −1 which does not make sense since you cannot have negative time
c 6 hours

3 4 mph

4 **a** It is equal to 20 when $d = 0$
b 99 m

5 80 km/h and 60 km/h

6 $8\frac{4}{7}$ minutes

7 1 m/s

Mixed practice

1 $-10 \pm 4\sqrt{7}$

2 $\frac{11 \pm \sqrt{13}}{6}$

3 − 4

4 18

5 $8 \pm 8\sqrt{3}$

6 **a** 2 and 3 **b** No **c** 2 and 4 **d** No

Review in context

1 **a** His speed upstream will be 2 km/h less due to the current; likewise his speed downstream will be 2 km/h greater.
b **i** $\frac{15}{x-2}$ **ii** $\frac{15}{x+2}$
c $\frac{15}{x-2} + \frac{15}{x+2} = 3$
d $(5 + \sqrt{29})$ km/h [10.39 km/h (2 d.p.)]

2 **a** 16 and 48 ohms **b** 3 and 6 ohms
c 3, 5 and 6 ohms **d** 4 and 12 ohms

3 3 km/h

4 **a** 22.5 cm **b** 2 cm **c** 1.83 cm (3 s.f.)

5 48.6 km/h (3 s.f.)

Unit 9 Answers

E9.1

You should already know how to:

1 Mean 8.8, standard deviation 5.44

2 2.29

3 a 5

b 2

c Mean 3.26, standard deviation 1.28

Practice 1

1 a $\sum x = 77$

b $\sum y = 109$

c $n = 5$

d $\sum xy = 1704$

e $\bar{x} = 15.4$

f $\bar{y} = 21.8$

g $\bar{x}\bar{y} = 335.72$

h $\sigma_x = 2.416609195$

i $\sigma_y = 2.13541565$

j correlation coefficient $= \dfrac{\frac{1}{5}(1704) - (335.72)}{(2.4166)(2.1354)}$

k 0.9844

2 $r = \dfrac{\frac{1}{10} \times 5313 - 17 \times 30}{s_x \times s_y}$

$s_x = \sqrt{\frac{1}{10} \times 3250 - 17^2} = \sqrt{36} = 6$

$s_y = \sqrt{\frac{1}{10} \times 9250 - 30^2} = \sqrt{25} = 5$

$\Rightarrow r = \dfrac{531.3 - 510}{6 \times 5}$

$= 0.71$

3 a Student's own scatter diagram showing the data in the question

b –0.455

c There is a negative correlation. The greater the satisfaction the students had for social media, the less satisfaction the students had for the television.

Mixed Review

1 a

	x	y	x^2	y^2	xy
	23	45	529	2025	1035
	25	51	625	2601	1275
	25	49	625	2401	1225
	26	54	676	2916	1404
	27	52	729	2704	1404
	29	60	841	3600	1740
	32	58	1024	3364	1856
	33	58	1089	3364	1914
Totals	220	427	6138	22 975	11 853

b Covariance

$s_{xy} = \frac{1}{n}\Sigma xy - \bar{x}\bar{y}$

$= \frac{1}{8}$ 11 853 – (27.5) (53.375)

$= 13.8125$

c $\sum \sigma_x = \sqrt{\dfrac{\sum x^2}{n} - \left(\dfrac{\sum x}{n}\right)^2}$

$= \sqrt{\dfrac{6138}{8} - \left(\dfrac{220}{8}\right)^2}$

$= 3.3166$

$\sum \sigma_y = 4.7942$

$\sum \sigma_x \sum \sigma_y = 15.9$

d $r = 0.868$

e There is strong positive correlation; the more screen time per week a student had, the more hours of sleep per week they had.

2 a 21.87

b 6.14

c r = 0.610. There is positive correlation; the more items that are sold, the higher the average price per item.

Unit 10 Answers

E10.1

You should already know how to:

1 x^7

2 a 125 b $\frac{1}{2}$ c 4

3 a $6^{\frac{1}{2}}$ b $3^{\frac{2}{3}}$ c $5^{\frac{3}{2}}$

4 $2 \times 1.18^{52} = 10\,937$ (to the nearest whole number)

Practice 1

1 a $\log 500 = x = 2.70$ b $\log 150 = x = 2.18$
c $\log 60 = x = 1.78$ d $\log_2 45 = x = 5.49$
e $\log_2 6 = x = 2.58$

2 a $8^2 = 64$ b $8^{\frac{2}{3}} = 4$ c $10^{-1} = 0.1$
d $5^5 = x$ e $b^4 = b$ f $x^0 = y$

3 a $\log_3 9 = 2$ b $\log_4 0.125 = -\frac{2}{3}$ c $\log_{1000} 10 = \frac{1}{3}$

4 a 4 b 5 c −2 d 5
e −5 f −3 g 0 h −2

5 a $\frac{1}{2}$ b $\frac{3}{2}$ c 2 d 4

6 a 4.09 b 2.68 c 3.13 d 0.77

7 a $x = 3.36$ (3 s.f.) b $x = 2.86$ (3 s.f.)
c $x = 3.20$ (3 s.f.) d $x = 6.49$ (3 s.f.)

8 a $\frac{1}{3}$ b 2 c $-\frac{1}{3}$ d 0 e $\frac{2}{5}$

Practice 2

1 2.32 2 0.661 3 1.7 4 0.232
5 0.58 6 2.58 7 4.2 8 4.3

Practice 3

1 a 303 900 and 307 850.7 ≈ 307 851
b $P(t) = 300\,000 \times 1.013^t$
c $350000 = 300000 \times 1.013^t$, year = 2012

2 t = 4.32, so 5 years

3

Time	Time period (in 5-hour intervals) *t* (hours)	Amount of caffeine (*C* mg)
08:00	$t = 0$	120
13.00	$t = 1$	60
18:00	$t = 2$	30
23:00	$t = 3$	15

a $r = 0.5$
b $C(t) = 120 \times 0.5^t$
c $C(7) = 120 \times 0.5^7 = 0.9375$
d $0.02 = 120 \times 0.5^t$, 13 hours (12.55)

4 $T = 72 \times 0.98^w$, $60 = 72 \times 0.98^w$, $w = 9.02$, thus after 10 weeks

5 a

Year	Population
0	100 000
1	90 000
2	81 000
3	72 900

b $P(t) = 100\,000 \times 0.9^t$
c $25\,000 = 100\,000 \times 0.9^t$, $t = 13.15$, thus after 14 years

Practice 4

1 18 242 years

2 a $b = -0.0866$ b $A = 3e^{-0.0866t}$ c 0.265

3 a 1000 b 2014 c 5.59 or approximately 6 weeks

4 a 121 rabbits b 5 years

5 a 50 wombats b 1.73 years or approximately 21 months

6 a $r = 0.0200$ b 6230 c 34 years

Mixed practice

1 a $\log_7 23 = x$ b $\log 95 = x$ c $\log_8 6 = x$
d $\log_4 47 = x$ e $\log_{12} 1200 = x$

2 a $5^3 = 125$ b $3^{-2} = \frac{1}{9}$ c $10^3 = 1000$
d $7^4 = 2401$ e $a^n = m$

3 a 4 b 2 c −1 d 0
e 1 f $\frac{1}{2}$ g $\frac{3}{2}$

4 a 2.81 b 1.79 c 3.57
d 1.39 e 2.81 f 1.61

5 a 2.73 b 1.86 c 3.00
d 1.46 e 0.92 f 0.65

6 a $-\frac{1}{2}$ b $\frac{7}{8}$ c $-\frac{1}{6}$

7 a $C(t) = 4 \times 1.055^t$
b $C(26) = 4 \times 1.055^{26} = \$16.09$
c 30 years, or in the year 2020

8 a 100°C b 54.9°C c 5 minutes

Review in context

1 a 3.4
b i 7.4 ii 5.27 iii 5.5 iv 5.9 v 5.9
c 91 488 815 microns
d 4280 microns

2 a 2.8 b 3.98×10^{-10} c 10^{-7}

3 a 4.2 b 0.005

4 a 110 dB, recommended b 64.9 db, not needed
c 169 dB, necessary

E10.2

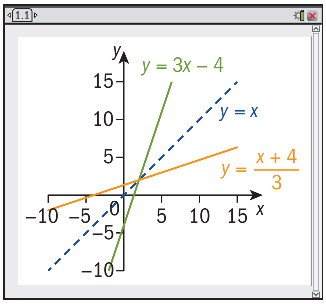
You should already know how to:

1 **a** 0.699 **b** 1.10 **c** 2.58

2 $x = 2.73$ **3** $x = -1$

4 $y = 3x - 4$

Inverse function: $y = \frac{x+4}{3}$



The functions are reflections in the line $y = x$.

5 $y^{-1} = 5^{\left(\frac{x+11}{2}\right)} - 3$

Practice 1

1 **a** log 12 **b** log 15 **c** log 200

d $\log \frac{z^{\frac{1}{2}} x^3}{y}$ **e** $\ln \frac{x^9}{8}$ **f** $\log \frac{x^3}{96y^4}$

2 **a** $\ln a + \ln b$ **b** $\ln a - \ln b$ **c** $2\ln a + \ln b$

d $\frac{1}{2}\ln a$ **e** $-2\ln a$ **f** $\ln a + \frac{1}{2}\ln b$

g $3\ln a - \ln b$ **h** $3\ln a - 2\ln b$ **i** $\frac{1}{2}(\ln a - \ln b)$

3 **a** $x + y$ **b** $y - x$ **c** $x - y$ **d** $2x$

e $2y$ **f** $3x$ **g** $y + 3x$

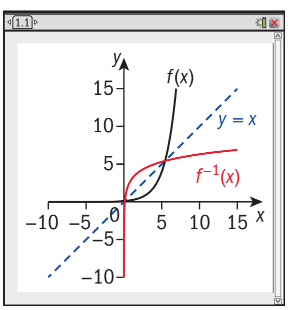
4 **a** $2x + 3y + 2$ **b** $5x - 4y - 2$

c $\frac{1}{3}(6x + 8y + 3)$ **d** $-x - \frac{1}{2}y - 1$

Practice 2

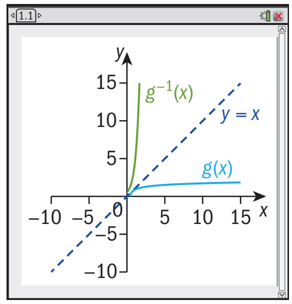
1 **a** $f^{-1}(x) = \log_2 x + 3$; Domain of f is $\mathbb{R}$; range of f is $\mathbb{R}^+$.

Domain of f^{-1} is $\mathbb{R}^+$; range of f^{-1} is $\mathbb{R}$.



b $g^{-1}(x) = \frac{e^{x^2}}{2}$, $x > 0$; Domain of g is $x > \frac{1}{2}$; range of g is $\mathbb{R}^+$.

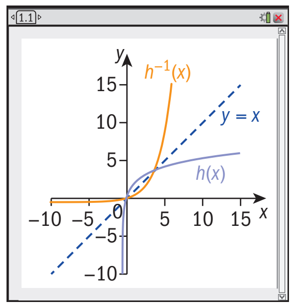
Domain of g^{-1} is $\mathbb{R}^+$ Range of g^{-1} is $y > \frac{1}{2}$.



c $h^{-1}(x) = \frac{10^{\frac{x}{4}} - 1}{2}$

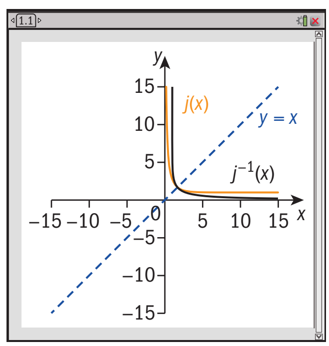
Domain of h is $x > -\frac{1}{2}$; range of h is $\mathbb{R}$. Domain of h^{-1} is $\mathbb{R}$;

range of h^{-1} is $y > -\frac{1}{2}$.



d $j^{-1}(x) = \frac{2 + e^x}{e^x - 1}$; Domain of j is $x > 1$; range of j is $\mathbb{R}^+$.

Domain of j^{-1} is $\mathbb{R}^+$; range of j^{-1} is $y > 1$.



2 $g(x) = \log x^n = n \log x$. $g(x) = n \log x$ is a vertical dilation of $f(x)$ scale factor n.

3 $f(x) = -\log_3 x + 2$. Reflect the graph of $y = \log_3 x$ in the x-axis and translate it vertically 2 units in the positive y direction.

4 **a** **i** $\log_2\left(\frac{8}{x}\right) = \log_2 8 - \log_2 x = 3 - \log_2 x$.

ii $\log_2(x+3)^2 = 2\log_2(x+3)$.

b **i** Reflect $y = \log_2 x$ in the x–axis then translate 3 units vertically in the positive y–direction.

ii Translate 3 units horizontally in the negative x–direction, and dilate vertically by scale factor 2.

Practice 3

1 **a** $\log p^4q^2$ **b** $\log \frac{p^n}{q^3}$ **c** $\ln\frac{(x-2)^3}{\sqrt{x}}$

d $\ln\frac{x}{(x-2)^2}$ **e** $\ln\left(\frac{e}{x}\right)$ **f** $\ln(x^4(x-1)^2)$

2 **a** $a - b$ **b** $a + b$ **c** $2a + c$

d $\frac{1}{2}(b-a)$ **e** $\frac{1}{2}c+b$

3 **a** $\ln(x-3)+\ln(x+3)$
b $\ln(x+2)+\ln(x+3)$
c $\ln(x-3)-\ln(x+4)$
d $\ln(x+1)-\ln(x+2)-\ln(x-2)$
e $\ln(x-6)+\ln(x+1)-\ln(x+2)-\ln(x-2)$
f $2\ln x+\ln(x+2)+\ln(x-5)-\ln(5x-20)$

Practice 4

1 **a** 2.81 **b** 1.29 **c** 2.10
d 0.732 **e** 1 **f** 1.16

Practice 5

1 **a** $\frac{3}{5}\ln x$ **b** $\frac{2}{5}\ln x$ **c** $\frac{1}{10}\ln x$ **d** $\frac{1}{5}(\ln x - \ln 3)$

2 **a** $\ln x^5$ **b** $-\ln x$ **c** $\ln x^{\frac{7}{2}}$ **d** $\ln\left(\frac{x^{\frac{1}{2}}}{9}\right)$

Practice 6

1 **a** $x = 2.93$ **b** $x = 1.91$ **c** $x = 1.30$

2 **a** $x = 0.693$ **b** $x = 0.792$ **c** $x = 0.545$

3 **a** $x = 0.396$ **b** $x = 0.417$ **c** $x = 1.04$

4 **a** $x = 0$ or $x = 2.58$
b $x = 1$ or $x = 1.29$

5 **a** $x = 1.2$ **b** $x = \frac{5}{34}$ **c** $x = -\frac{5}{8}$
d $x = 2$ or $x = -7$. $x = -7$ is not possible, hence $x = 2$
e $x = -4$ or $x = 5$. $x = -4$ is not possible, hence $x = 5$

6 **b** 10 *years*

7 **a** 15 is the initial number of video viewers.
b 21 hours
c 23 hours
d Students' own answers

8 **a** $x = 0.747$ **b** $x = 1.35$

c $x = \frac{1}{3}$ **d** $x = 2$

Mixed practice

1 **a** $\log a + \log b + \log c$ **b** $\log a + \log c - \log b$

c $3\log a + 2\log b + 4\log c$ **d** $\log a + \frac{1}{2}\log b$

e $\log a + \frac{1}{2}\log b - \log c$

2 **a** log 12.5 **b** ln 250 **c** $\log_4 40$

d $\log_a 4$ **e** $\ln\left(\frac{x^6}{(x-1)^2(2x+6)^3}\right)$ **f** $\log_4 54$

g $\log_7\frac{4x^{\frac{1}{3}}}{y^2}$

3 **a** $-y$ **b** $x + 2y$ **c** $-(x + y)$
d $2x + 2y$ **e** $3x - 2y$

4 **a** $5x + y$ **b** $1 + \frac{1}{4}(x + 3y)$

c $2(3x + 4y - 2)$ **d** $1 + 7x - \left(\frac{1}{2}\right)y$

5 **a** $f^{-1}(x) = \frac{\ln x - \ln 30}{2\ln 5}$; Domain of f is $\mathbb{R}$; range of f is $\mathbb{R}^+$.
Domain of f^{-1} is $\mathbb{R}^+$; range of f^{-1} is $\mathbb{R}$.

b $g^{-1}(x) = 10^{(x+2)^3}$; Domain of g is $\mathbb{R}^+$; range of g is $\mathbb{R}$;
Domain of g^{-1} is $\mathbb{R}$; range of g^{-1} is $\mathbb{R}^+$.

c $h^{-1}(x) = \frac{e^{-\frac{3}{2}x}+1}{3}$

Domain of h is $x > \frac{1}{3}$; range of h is $\mathbb{R}$.
Domain of h^{-1} is $\mathbb{R}$; range of h^{-1} is $y > \frac{1}{3}$.

d $k^{-1}(x) = \frac{5 \cdot 4^x}{7 - 4^x}$; Domain of k is $\mathbb{R}^+$; range of k is
$k(x) < \log_4(7)$.
Domain of k^{-1} is $\mathbb{R}$, $x < \frac{\ln 7}{\ln 4}$; range of k^{-1} is $\mathbb{R}^+$.

6 **a** $y = 3 + \log_3(x)$; Translate the graph of $y = \log_3 x$ by 3 units vertically in the positive y–direction.

b $y = 4\log_3(x - 9)$; Dilate $y = \log_3 x$ vertically by scale factor 4 and translate horizontally 9 units in the positive x-direction.

7 **a** $c - b$ **b** $a + b + c$ **c** $3a - 2c$

d $\frac{1}{2}(2b - (a + c))$ **e** $\frac{1}{2}(3a + b)$

8 **a** $\ln(x-5)+\ln(x+5)$
b $\ln(x-7)+\ln(x+4)$
c $\ln(x+1)-\ln(x-8)$
d $\ln(x-5)+\ln(x+1)-\ln(x+4)-\ln(x+2)$
e $\ln(x-12)-\ln x-\ln(x-2)$

9 **a** $x = \pm 1$ **b** $x = 2$

10 **a** $x = 15$ **b** $x = 112$ **c** $x = 3$
d $x = 2$ **e** $x = 6$

11 **a** $\frac{x}{y}$ **b** $3x + y$ **c** $4x + 2y$

d $y - x$ **e** $x - y$

12 **a** $x = 0.235$ **b** $x = -4.64$ **c** $x = 1.66$ **d** $x = 2.06$

13 **a** $\frac{y}{x}$ **b** $\frac{x}{y}$ **c** $\frac{2y}{x}$

14 **a** $\frac{3}{8}\ln x$ **b** $\frac{3}{8}\ln x - 16$

15 **a** 24 **b** 5

16 **a** 52 days **b** 2.8 million

17 $y = 15e^{0.05x}$

Review in context

1 **a** 8.90
b Scale factor of 6.31 times bigger
c Add 1.3
d 40 times bigger
e Eastern Sichuan was 40 times bigger.

2 **a** 6.8 acidic **b** 2.8 **c** 6.2
d Yes, it is 7.1 **e** $1 \times 10^{-7} - 3.98 \times 10^{-8}$

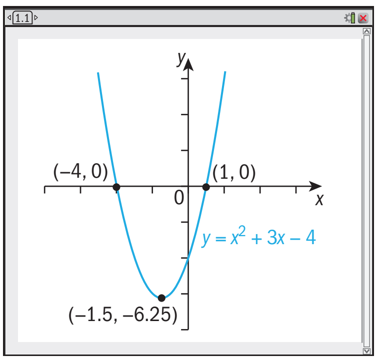
3 **a** 137 dB **b** 6.3 times bigger

E10.3

You should already know how to:

1 $x < 7$

2 **a**



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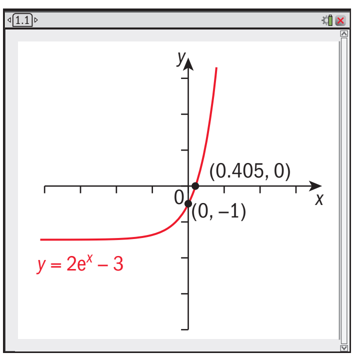

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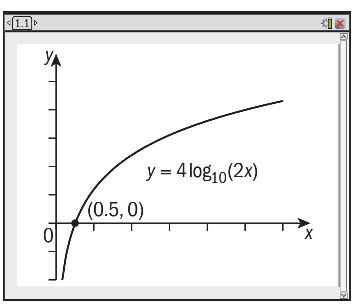

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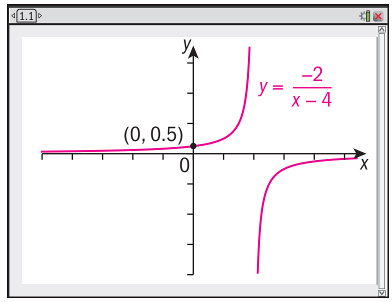

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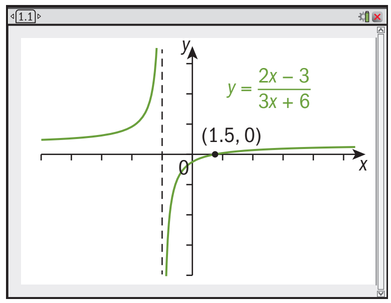

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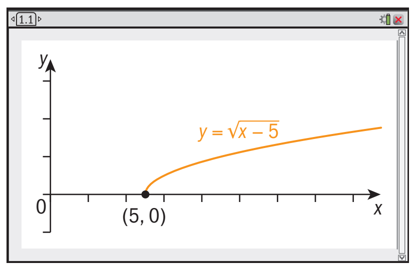


3 $x = -3 \pm \sqrt{6}$

Practice 1

1 **a** $x < -2$ or $x > 6$ **b** $-1.5 \le x \le 2$
c $x \le -0.4$ or $x \ge 5$ **d** $-4 < x < 1.5$
e $-\frac{3}{2} < x < \frac{2}{3}$

2 **a** $h(t) > 9600$ m
b About 23 seconds

3 $x(x + 6) > 216$; $x > 12$ cm; $(x + 6) > 18$ cm

4 $x(12 - x) \le 24$; $0 \text{ m} < x \le 2.53$ m or $9.47 \text{ m} \le x < 12$ m

5 $y = 0.005x^2 - 0.23x + 22 > 25$; $x > 56$ years

6 **a** $h(t) > 0$ **b** 3.56 seconds

7 $x > 2.73$ cm

8 4.47 cm × 4.47 cm × 2 cm

Practice 2

1 a $x > 1$ or $-1 < x < 0$

b $0 < x \le \frac{1}{4}$

c $-1 < x < 4$

d $x > 5.09$

e $-1.5 < x < -1$ or $x > 1.57$

f $-1.5 < x < -1$ or $x < -5$

g $-3 < x < 0$

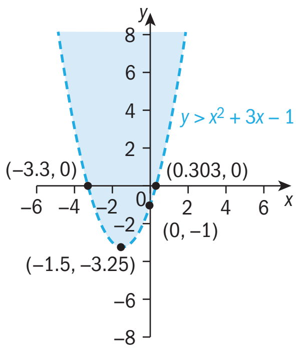
h $-4 \le x \le -2$ or $1 \le x \le 6$

2 a $R = \frac{2R_1}{2+R_1} \Rightarrow \frac{2R_1}{2+R_1} > 1,\ R_1 > 2$

b $R_2 > 7.48$

Practice 3

1 a



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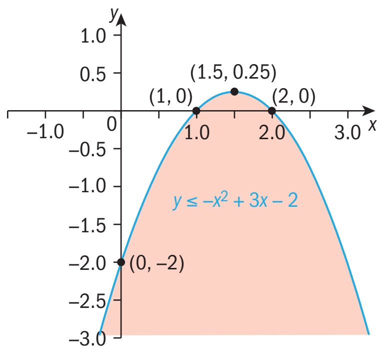

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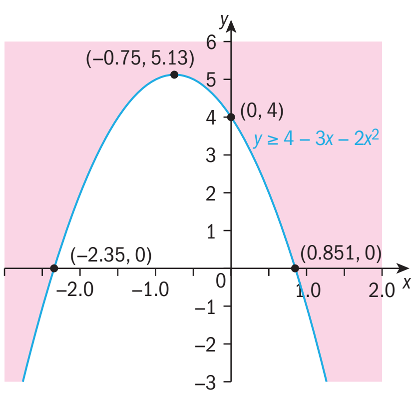

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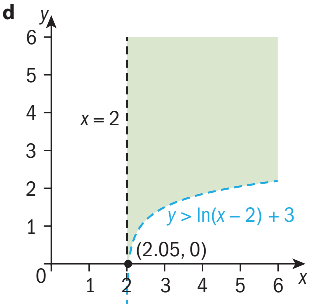

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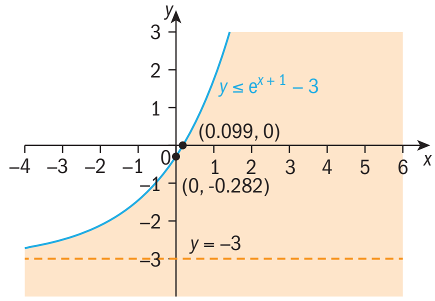

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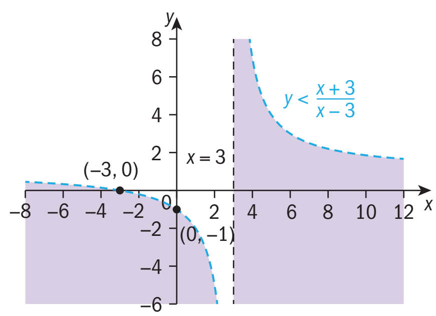


2 a $y < -x^2 - 4x + 5$

b $y \ge 0.5x^2 + 2x - 1$

c $y \le e^{2x+1} - 4$

d $y > \frac{x}{x-2}$

3 a i $f(x) = x^2 - 4$ **ii** $y > f(x)$

b i $f(x) = -\frac{1}{3}(x-1)^2 + 3$ **ii** $y > f(x)$

c i $f(x) = x^2 - 6x + 5$ **ii** $y < f(x)$

4 a $f(x) = -\frac{3}{2025}(x - 45)^2 + 3$

b Domain $\{x \mid 0 \le x \le 90\}$, range $\{y \mid 0 \le y \le 3\}$

c $0 \le y \le f(x)$

Practice 4

1 a $-1 \le x \le 3$

b $x < 2$ or $x > 8$

c $-3 \le x \le 4$

d $-10 \le x \le 4$

e $-3 < x < \frac{1}{2}$

f $x < -\frac{1}{3}$ or $x > 1$

g $-\frac{1}{2} < x < \frac{2}{3}$

2 $0 < x \le 1.70$ cm

3 Student's own answers

Practice 5

1 **a** $x < 0.209$; $x > 4.79$
b $-6.32 < x < 0.317$
c $-2.19 \le x \le 0.686$
d $x < 0.333$ or $x > 2$
e $-2.79 \le x \le 1.79$
f $x \le -0.414$ or $x \ge 2.41$
g $-0.290 < x < 0.690$
h $-9.12 \le x$ or $x \ge -0.877$
i $x < -1.18$ or $x > 0.425$
j $0.209 < x < 4.79$

2 **a** $\Delta = 0$, hence only one root, $x = -1$. Since the graph lies entirely above the x-axis except for this one value, there is only solution, $f(x) = 0$ for $x = -1$.
b $\Delta = 0$, hence only one root, $x = 2$. Since the graph lies entirely above or on the x-axis, the solution is $x \in \mathbb{R}$.
c Since $\Delta < 0$, the quadratic has no roots. Since $a > 0$, its graph is always above the x-axis for all values of x. Hence $x \in \mathbb{R}$.
d Since $\Delta < 0$, the quadratic has no roots. Since $a < 0$, its graph is always below the x-axis for all values of x. Hence $x \in \mathbb{R}$.

Practice 6

1 **a** $x < -1$ or $x > 1$
b $-1 < x \le 4$
c $x < -1$ or $x > 4$
d $x \ge 3$ or $x < 2$
e $0 < x < \frac{1}{4}$
f $x > 5$ or $x \le -\frac{2}{3}$

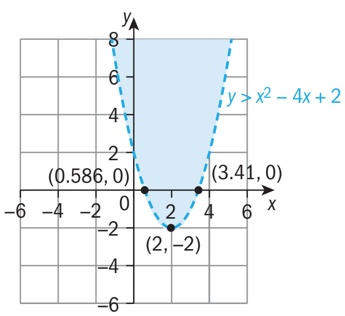
2 $-3 < x < -1$

3 17

Mixed practice

1 **a** $x < -3$ or $x > -1$
b $x \le -8.58$ or $x \ge 0.583$
c $x < -3.59$ or $x > 2.09$
d $x > 0$ or $x < -3$
e $0 < x < \frac{2}{7}$
f $\frac{10}{3} < x < 5$

2 **a**



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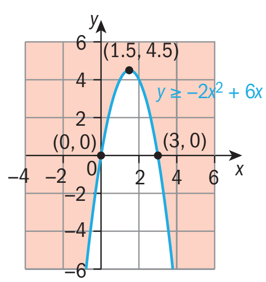

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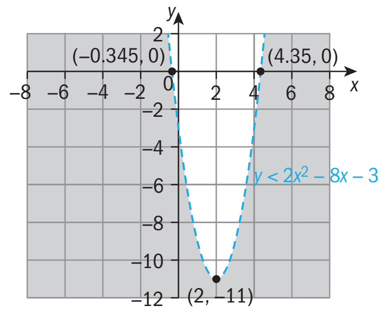

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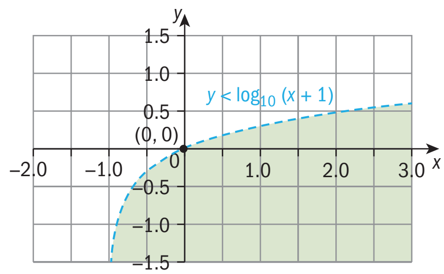

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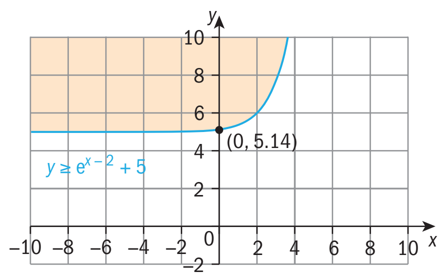

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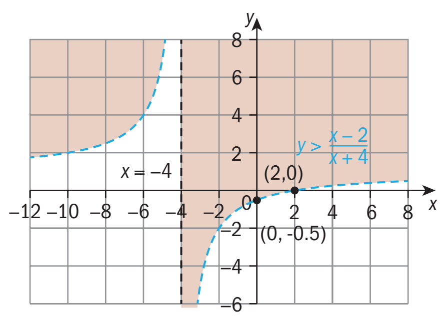


3 **a** $-6 < x < 1$
b $x < -2$ or $x > 12$
c $-7 < x < 5$
d $-8 < x < 2$
e $0 < x < 5.5$
f $x \le \frac{3}{5}$ or $x \ge 2$
g $x > 3$ or $x < -5$
h $-1 < x < 1$

4 36.1 meters

5 Between 12 and 28 tourists

6 4.7 cm < radius < 9.6 cm; 6.9 cm < height < 29.8 cm

7 1.66 in $< x <$ 6.15 in

Review in context

1, 2 Student's own answers.

Unit 11 Answers

E11.1

Practice 1

1 Eulerian circuits: c, d,
Eulerian trails: a, b, e, h, i
Non-traversable: f, g

2 **a** BE **b** CD **c** DF
d BE **e** AC **f** AE

3 **a** BE **b** CD **c** DF
d BE **e** AC **f** AE

3 **a** Traversable, e.g. CACBAB
b Non-traversable
c Traversable, e.g. EAEDCDBCBA
d Traversable, e.g. EBABCADCED
e Traversable, e.g. ADABCBCD
f Non-traversable

4 Monday: She can deliver all the papers without retracing her steps, but then will need to walk back to the shop.
Tuesday: She can deliver all the papers and return to the shop without having to walk the same street twice.
Wednesday: She will have to walk down some unnecessary roads or retrace her steps before distributing all the newspapers.

Practice 2

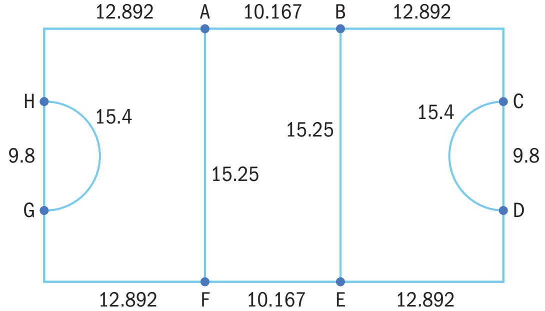
1 Double AB, CD & DE. ABACDEDCBEFA; 164 minutes

2 **a** There is only one route to and from A, so she will need to walk along it twice.
b CD, DE, EG
c 76 minutes

3 Double BE + EI; GL + LK. Total distance = 12 600 m.

4 **a** 17 700 m (17.7 km)
b 20 600 m (20.6 km)

5 **a** 15.4 m
b 2.725 m
c



d All eight vertices are odd.
e CD, GH = 9.8
AB, EF = 10.167
f Total weight = 152.802; with doubling: 192.736 so 193 m (3 s.f.).
e.g. AHGHGFABEFEDCDCBA. Since four edges need doubling, no solution will be shorter than doubling the shortest four edges.

Mixed practice

1

Graph	Connected	Complete	Has an Eulerian circuit	Has an Eulerian trail	Non-traversable
a	Y	Y	N	N	Y
b	Y	N	N	N	Y
c	Y	Y	Y	Y	N
d	Y	N	N	Y	N
e	Y	N	N	Y	N
f	N	N	N	N	Y
g	Y	N	N	Y	N
h	Y	N	N	N	Y
i	N	N	N	N	Y

2 **a** $\sqrt{3}$
b Design A uses more plastic.
c A has an Eulerian circuit because all the nodes are even. B has four odd nodes.
d A: 11.2 cm. B: 11.5 cm
e A requires less movement.

3 18.7 + 1.4 (AC) + 1.7 (GH) = 21.8 km

4 **a** ABEGFCDA
b ACDEGFBA
c ACBFDGEA
d ACBDHEGFA

5 ABDCA = 270

Review in context

1 **a** Start/finish at D and E because they are odd nodes. 225 m
b She will now need to duplicate a route, and the shortest to duplicate is the 15 m tunnel from D to E. She should now start and finish at A and B, since they are odd nodes. The new route will be 220 m.
c CDE(20)D(15)E(15)ADBABC. 245 m

2 **a** A Hamiltonian path connects all the nodes and hence would allow information to pass between all the buildings. Since data can pass from one building to another via a third, it is not necessary to use any additional cabling.
b ABCDE(15). 70 m

E11.2

You should already know how to:

1 **a** $\sqrt{53} = 7.28$ (3 s.f.) **b** 5

2 $x = 0, y = 1$

3 94.1°

Practice 1

1 a i $\begin{pmatrix}2\\4\end{pmatrix}$ **ii** $\begin{pmatrix}4\\-3\end{pmatrix}$ **iii** $\begin{pmatrix}-2\\-2\end{pmatrix}$

iv $\begin{pmatrix}2\\-4\end{pmatrix}$ **v** $\begin{pmatrix}2\\-4\end{pmatrix}$ **vi** $\begin{pmatrix}5\\-2\end{pmatrix}$

b i $\overrightarrow{AD}=\begin{pmatrix}1\\-5\end{pmatrix}$ $\overrightarrow{DA}=\begin{pmatrix}-1\\5\end{pmatrix}$

ii $\overrightarrow{EG}=\begin{pmatrix}1\\6\end{pmatrix}$ $\overrightarrow{GE}=\begin{pmatrix}-1\\-6\end{pmatrix}$

c The components have the same size, but opposite sign.

d i $\begin{pmatrix}1\\1\end{pmatrix}$ **ii** $\begin{pmatrix}3\\5\end{pmatrix}$ **iii** $\begin{pmatrix}5\\-2\end{pmatrix}$ **iv** $\begin{pmatrix}0\\0\end{pmatrix}$

e i F **ii** G **iii** D

2 a *C* to *A* **b** *E* to *C* **c** *A* to *D* **d** *H* to *B*

3 a (5, 9) **b** (1, 8) **c** (0, 3) **d** (14, –5)

4 a $\overrightarrow{MN}=\begin{pmatrix}3\\2\end{pmatrix}$ **b** $\overrightarrow{PQ}=\begin{pmatrix}7\\-3\end{pmatrix}$

c $\overrightarrow{RS}=\begin{pmatrix}-4\\7\end{pmatrix}$ **d** $\overrightarrow{TU}=\begin{pmatrix}-1\\-4\end{pmatrix}$

5 $\overrightarrow{DC}=\begin{pmatrix}2\\-4\end{pmatrix}$

6 b $\begin{pmatrix}3\\1\end{pmatrix}$

7 E.g. (0, 0) and (–5, 3); (3, 1) and (–2, 4); (6, 11) and (1, 14)

Practice 2

1 a 25 **b** 5 **c** 10

2 a 5 **b** $\sqrt{10}$ **c** $2\sqrt{5}$ **d** $5\sqrt{2}$ **e** 13 **f** $\sqrt{281}$

3 a 6.40 **b** 8.25 **c** 4.47 **d** 13.2 **e** 8.94 **f** 12.5

4 a 6 **b** 26 **c** 25 **d** 1 **e** 10 **f** 5

5 a 9.06 **b** 3.16 **c** 13.3 **d** 6.80 **e** 20.8 **f** 21.0

6 $|AB| = 5\sqrt{2} = |CD|$

7 Any four from (8, 1), (8, –5), (9, 0), (9, –4), (4, 1), (4, –5), (3, 0), (3, –4)

8 $x = 2, y = 1$

9 $x = 2$

Practice 3

1 a $\begin{pmatrix}1\\5\end{pmatrix}$ **b** $\begin{pmatrix}8\\-1\end{pmatrix}$ **c** $\begin{pmatrix}-5\\17\end{pmatrix}$ **d** $\begin{pmatrix}0\\0\end{pmatrix}$ **e** $\begin{pmatrix}-6\\-4\end{pmatrix}$

2 a $\overrightarrow{AB}=\begin{pmatrix}-1\\-7\end{pmatrix}$ $\overrightarrow{BA}=\begin{pmatrix}1\\7\end{pmatrix}$

b $\overrightarrow{AB}=\begin{pmatrix}5\\-1\end{pmatrix}$ $\overrightarrow{BA}=\begin{pmatrix}-5\\1\end{pmatrix}$

c $\overrightarrow{AB}=\begin{pmatrix}-11\\10\end{pmatrix}$ $\overrightarrow{BA}=\begin{pmatrix}11\\-10\end{pmatrix}$

d $\overrightarrow{AB}=\begin{pmatrix}9\\0\end{pmatrix}$ $\overrightarrow{BA}=\begin{pmatrix}-9\\0\end{pmatrix}$

e $\overrightarrow{AB}=\begin{pmatrix}-9\\-7\end{pmatrix}$ $\overrightarrow{BA}=\begin{pmatrix}9\\7\end{pmatrix}$

3 (–15, –4)

4 a $a = 0, b = -1$ **b** $a = 1, b = -2$ **c** $a = 3, b = 3$

Practice 4

1 $a = 3$ **2** $b = 8$ **3** $OM=\begin{pmatrix}10.5\\5.5\end{pmatrix}$

4 b $|AB| = \sqrt{17}$ and $|CD| = 2\sqrt{17}$. They are different lengths.

6 a $\overrightarrow{AB} = \overrightarrow{DC} = 10\mathbf{u} + 5\mathbf{v}$

7 *ABCD* or *ABDE*

Practice 5

1 a 127° **b** 67° **c** 25° **d** 35°

3 b 111°

4 37°

5 $m = 4$

6 a $ab + (a + 2)(2 - b) = 0$, so $a - b = -2$

b It is an equation in two unknowns.

c $a = -\frac{1}{2}, b = \frac{3}{2}$

7 Any non–zero multiple of $\begin{pmatrix}1\\4\end{pmatrix}$

8 a $OC=\begin{pmatrix}2\\-3\end{pmatrix}$ **b** 14.5°

9 –8

10 LHS = RHS = 16

Practice 6

1 a $-\mathbf{u}+\mathbf{v}$ **b** $-\mathbf{v}-\mathbf{w}$ **c** $-\mathbf{u}+\mathbf{v}+\mathbf{w}$

2 a $\mathbf{b}-\mathbf{a}$ **b** $\mathbf{b}-\mathbf{a}$ **c** $\mathbf{b}-\mathbf{a}$ **d** $\mathbf{b}-2\mathbf{a}$ **e** $\mathbf{a}+\mathbf{b}$

3 a $\mathbf{r} = 5\mathbf{q} - 4\mathbf{p}$

4 $\mathbf{s} = 3\mathbf{t} - 2\mathbf{u}$

5 a $\mathbf{b} + \frac{3}{2}\mathbf{a}$

c A rectangle with sides in the ratio 2 : 3

Mixed practice

1 a $\begin{pmatrix}5\\4\end{pmatrix}$ **b** $\begin{pmatrix}-2\\3\end{pmatrix}$ **c** $\begin{pmatrix}3\\4\end{pmatrix}$ **d** $\begin{pmatrix}-11\\-19\end{pmatrix}$

2 a $\overrightarrow{AB}=\begin{pmatrix}-3\\-3\end{pmatrix}$ $\overrightarrow{BA}=\begin{pmatrix}3\\3\end{pmatrix}$

b $\overrightarrow{AB}=\begin{pmatrix}-7\\3\end{pmatrix}$ $\overrightarrow{BA}=\begin{pmatrix}7\\-3\end{pmatrix}$

c $\overrightarrow{AB}=\begin{pmatrix}3\\-5\end{pmatrix}$ $\overrightarrow{BA}=\begin{pmatrix}-3\\5\end{pmatrix}$

d $\overrightarrow{AB}=\begin{pmatrix}13\\-14\end{pmatrix}$ $\overrightarrow{BA}=\begin{pmatrix}-13\\14\end{pmatrix}$

3 a $\begin{pmatrix}-3\\-3\end{pmatrix}$ **b** $\begin{pmatrix}9\\-6\end{pmatrix}$ **c** $\begin{pmatrix}3\\-5\end{pmatrix}$ **d** $\begin{pmatrix}-2\\15\end{pmatrix}$

4 a $a = 0, b = 1$ **b** $a = 2, b = -3$
c $a = 5, b = -4$ **d** $a = -2, b = -3$

5 a $\sqrt{29}$ **b** 5 **c** $3\sqrt{41}$ **d** $5\sqrt{17}$

6 a $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ **b** $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$ **c** $\begin{pmatrix} -5 \\ -3 \end{pmatrix}$

7 a 87° **b** 84° **c** 97°

8 $\begin{pmatrix} -6 \\ 20 \end{pmatrix}$ and $\begin{pmatrix} 9 \\ -30 \end{pmatrix}$ $\begin{pmatrix} 16 \\ 20 \end{pmatrix}$ and $\begin{pmatrix} 36 \\ 45 \end{pmatrix}$

$\begin{pmatrix} 12 \\ -18 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$ $\begin{pmatrix} 15 \\ 25 \end{pmatrix}$ and $\begin{pmatrix} -9 \\ -15 \end{pmatrix}$

$\begin{pmatrix} 5 \\ 8 \end{pmatrix}$ is the odd one out.

10 $a = -1$ or $a = 4$

11 $b = -3$

13 a $\overrightarrow{AB}$ is parallel to $\overrightarrow{OC}$ because $\overrightarrow{OC} = 2\overrightarrow{AB}$.

b $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \mathbf{b} + \mathbf{a} = \mathbf{a} + \mathbf{b}$

c $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = 2\mathbf{a} - \mathbf{b}$

d Since X lies on $\overleftrightarrow{OB}$, $\overrightarrow{OX}$ is parallel to $\overrightarrow{OB}$,

hence $\overrightarrow{OX} = k\overrightarrow{OB} = k(\mathbf{a} + \mathbf{b})$.

e Since X lies on $\overleftrightarrow{AC}$, $\overrightarrow{AX}$ is parallel to $\overrightarrow{AC}$,

hence $\overrightarrow{AX} = m\overrightarrow{AC}$.

So $\overrightarrow{OX} = \overrightarrow{OB} + \overrightarrow{AX} = \mathbf{b} + m(2\mathbf{a} - \mathbf{b})$

f $k(\mathbf{a} + \mathbf{b}) = \mathbf{b} + m(2\mathbf{a} - \mathbf{b})$

$m = \frac{1}{3}$, $k = \frac{2}{3}$, so ratio is 1 : 2

Review in context

2 a 76 cm **b** 45 cm **c** 67 cm
d 95 cm **e** 292 cm **f** 447 cm

3 a $15\ \text{m}^2$

b $\begin{pmatrix} -25 \\ -60 \end{pmatrix}$

c $250 + 600 + 650 = 1500 = 15$ m

4 a $\overrightarrow{AE} = \begin{pmatrix} 36 \\ 55 \end{pmatrix}$, 657 cm

b $\overrightarrow{OF} = \begin{pmatrix} 22 \\ -30 \end{pmatrix}$, $\overrightarrow{CF} = \begin{pmatrix} 24 \\ -45 \end{pmatrix}$, $CF = 510$ cm

c $\overrightarrow{AC} = \begin{pmatrix} 12 \\ 45 \end{pmatrix}$, $|\overrightarrow{AC}| = \sqrt{2169} = 3\sqrt{241}$

$\overrightarrow{CE} = \begin{pmatrix} 24 \\ 10 \end{pmatrix}$, $|\overrightarrow{CE}| = 26$

AC is longer

d Perimeter 18.2 m, area $18.6\ \text{m}^2$

5 a $\overrightarrow{PQ} = \begin{pmatrix} -24 \\ 32 \end{pmatrix}$

$\overrightarrow{PS} = \begin{pmatrix} 60 \\ 45 \end{pmatrix}$

b $\overrightarrow{PQ} \times \overrightarrow{PS} = \begin{pmatrix} -24 \\ 32 \end{pmatrix} \times \begin{pmatrix} 60 \\ 45 \end{pmatrix} = 0$ Hence walls PQ and PS are perpendicular.

c $\overrightarrow{SR} = \begin{pmatrix} -25 \\ 30 \end{pmatrix}$

d $\overrightarrow{SR}$ and $\overrightarrow{PQ}$ are not scalar multiples of each other, so $\overrightarrow{SR}$ is not parallel to $\overrightarrow{PQ}$. Not rectangular, opposite sides not parallel.

6 a $\overrightarrow{OU} = \begin{pmatrix} 18 \\ 22 \end{pmatrix}$, so $\overrightarrow{UV} = \begin{pmatrix} 22 \\ -22 \end{pmatrix}$, $\overrightarrow{WX} = \begin{pmatrix} 47 \\ -47 \end{pmatrix}$

b $\overrightarrow{UV}$ is parallel to $\overrightarrow{WX}$. $\overrightarrow{VW} = \begin{pmatrix} -25 \\ -25 \end{pmatrix}$ and $\overrightarrow{UX} = \begin{pmatrix} -50 \\ 0 \end{pmatrix}$ which are not parallel. Hence UVWX has exactly one pair of parallel sides, and so is a trapezoid.

c $\overrightarrow{VW} \cdot UV = 0$, $\overrightarrow{VW} \cdot \overrightarrow{XW} = 0$

d $|\overrightarrow{VW}| = 25\sqrt{2}$, $|\overrightarrow{UV}| = 22\sqrt{2}$. $|\overrightarrow{XW}| = 47\sqrt{2}$

Area = $17.25\ \text{m}^2$

e 135°